Problems (1)

State the order and degree of the given DE:

1. $y' = 4y$
2. $y'^3 - xy = 0$
3. $y''' + xy' = x \sin x$
4. $y^{(5)} - y^3 = y^4 = x^4 y''^2$

Verify that the given function is the general solution of the given DE, and hence find a solution satisfying the given initial conditions:

5. $y' + y \tan x = 0; \quad y = C \cos x; \quad y(\pi/3) = 5$
6. $yy' = -x; \quad x^2 + y^2 = C; \quad y(2) = 1$
7. $y'' - k^2 y = 0; \quad y = C_1 e^{kx} + C_2 e^{-kx}; \quad y(1) = 0, y'(1) = 2$
8. $y''' - 2y'' - y' + 2y = 6; \quad y = C_1 e^x + C_2 e^{-x} + C_3 e^{2x} + 3; \; y(0) = 3,$
   $y'(0) = 8, \quad y''(0) = 6$

Problems (2)

Find the general solution of the given DE:

1) $xy\,dx + (x+1)\,dy = 0$
2) $y' + y \tan x = 0$
3) $xy' + y = y^2$
4) $2y + (xy + 3x)y' = 0$
5) $(x^2 - 1)y' + 2xy^2 = 0$
6) $y' - xy^2 = 2xy$
7) $y' = \cos(y - x)$
8) $y' - y = 2x - 3$

9) $(x + 2y)y' = 1$

10) $(x - y)\, dx + (x + y)\, dy = 0$

11) $y' = \sqrt{4x + 2y - 1}$

12) $xy' = \sqrt{x^2 - y^2}$

13) $xy' - xe^{-y/x} = y$

14) $y' = \dfrac{x + 2y - 4}{2x - y - 3}$

15) $(y + 2)\, dx = (2x + y - 4)\, dy$

16) $y' = \dfrac{2x + y + 1}{4x + 2y - 3}$

17) $y' = 2\left(\dfrac{y + 2}{x + y - 1}\right)^2$

18) $(y' + 1)\ell n \dfrac{y + x}{x + 3} = \dfrac{y + x}{x + 3}$

Solve the given initial-value problem:

19) $(x + 1)y' = x\sqrt{y + 1}$; $y(0) = 0$

20) $y' = \dfrac{x + 3y}{3y + x}$; $y(1) = 0$

21) $(y - x - 1)y' = y - x$; $y(-5) = 5$

22) $(x^2 + 1)y' = y^2 + 1$; $y(0) = 1$

Solve the given DE by using the indicated change of variables

23) $xy' = e^{-xy} - y$; $z = xy$

24) $y' = (x + e^y - 1)e^{-y}$; $z = x + e^y$

Problems (3)

Show that the given DE is exact, and hence find its general solution:

(1) $(e^y + y\cos x)\, dx + (xe^y + \sin x)\, dy = 0$

(2) $x(x^2 + 2y^2)\, dx + y(2x^2 + y^2)\, dy = 0$

(3) $(x + y)\, dx + (x + y^2)\, dy = 0.$

(4) $Yx^{y-1}\, dx + x^y \ln x\, dy = 0$

Show that the given DE admit an integrating factor $\mu(x)$ or $\mu(y)$, then solve it:

(5) $(x^2 + 2y)\, dx - x\, dy = 0$

(6) $(xe^{x} + x\ell ny + y)dx + (\frac{x^2}{y} + x\ell nx + x\sin y)dy = 0$

(7) $2y^2(x + y^2)\,dx + xy(x + 6y^2)dy = 0$

(8) $Y(2y - x - 2xy)\,dx + (x + 4xy + 1)\,dy = 0$

(9) *Show that the DE*

$$\left(x\cos x + \frac{y^2}{x}\right)dx - \left(\frac{x\sin x}{y} + y\right)dy = 0$$

has an integrating factor of the form $\mu(xy)$, and hence solve it.

(10) *Show that the DE*

$(4x^2 + 2xy + 6y)\,dx + (2x^2 + 9y + 3x)\,dy = 0$

has an integrating factor of the form $(x + y)^n$ for some constant n, and hence solve it.

Solve the given DE:

(11) $\frac{2x(1-y)}{1+x^2}dx - \ell n(1+x^2)dy = 0$

(12) $(x^2 - x^2y - y^3)\,dx + (x^3 + xy^2)\,dy = 0$

(13) $\frac{y}{x}dx + (y^2 - \ell nx)dy = 0$

(14) $(x\cos y - y\sin y)\,dy + (x\sin y + y\cos y)\,dx = 0$

Problem (4)

(1) $xy' - 2y = 2x^4$

(2) $(2x + 1)y' = 4x + 2y$

(3) $y' + y \tan x = \sec x$

(4) $x(y' - y) = e^x$

(5) $y' = 2x(x^2 + y)$

(6) $(x + y^2)\, dy = y\, dx$

(7) $y' = \dfrac{y}{3x - y^2}$

(8) $y' - y \tan x = y^4 \cot x$

(9) $xy' - 2x^2\sqrt{y} = 4y$

(10) $xy\, dy = (y^2 + x)\, dx$

(11) $(x - 1)\, y' + xy = (x - 1)\, e^x$; $y(0) = 0$

(12) $2xy' + y = 2x^2\,(x + 1)\, y^3$; $y(1) = 1$

(13) $(xy - x)\, dx + (1 - x^2)\, dy = 0$; $y(0) = 2$