



### \* Dimension in units:-

→ 7 basic units

	S.I	B.S
1) mass (M)	kg	slug (Lbm)
2) Length (L)	m	ft (in)
3) Time (T)	s	s
4) Temp (Θ)	K [C°]	R [F]
5) amount of matter (N)	mole	mole
6) intensity of electrical current (i)	Ampere	Ampere
7) intensity of Light (C)	candela	candela

$$1 \text{ Feet} = 12 \text{ in.}$$

$$= 0.3048 \text{ m}$$

$$1 \text{ in} = 2.54 \text{ cm}$$

$$1 \text{ slug} = 14.59 \text{ kg}$$

$$1 \text{ Lbm} = 0.453 \text{ kg}$$

$$g = 9.806 \frac{\text{m}}{\text{s}^2} \text{ (S.I.)}$$

$$= 32.174 \frac{\text{ft}}{\text{s}^2} \text{ (B.S.)}$$

→ Derived units:- can be obtained using certain eqn.

$$\text{velocity} = \frac{\text{distance}}{\text{Time}}$$

$$[V] = \frac{L}{T} = \frac{\text{m}}{\text{s}}, \frac{\text{ft}}{\text{s}}, \frac{\text{in}}{\text{s}}$$

$$[V] = L^3 = \text{m}^3, \text{ft}^3, \text{in}^3$$

$$[P] = \frac{\text{mass}}{\text{Volume}} = \frac{\text{kg}}{\text{m}^3}, \frac{\text{slug}}{\text{ft}^3}, \frac{\text{Lbm}}{\text{in}^3}$$



$$\text{velocity} = \frac{ds}{dt} \Rightarrow \frac{[S]}{[t]} = \frac{L}{T}$$

$$\text{acceleration} = \frac{dw}{dt} = \frac{ds^2}{dt^2} \Rightarrow [a] = \frac{[S]}{[t^2]} = \frac{L}{T^2}$$

⇒ For integration:-

$$\int F \cdot dy = [F][y]$$

$$\text{Force} = F = ma$$

$$[F] = [m][a] = \frac{M \cdot L}{T^2} = \frac{kg \cdot m}{s^2}$$

\* Dimensional homogeneity. التجانس البعدي  
{تنسيق}

in the primary dimension of each terms for given eqn. is the same the eqn. is set to be dimensionally homogeneous.

\* Grid method:-

Cancellation of unit.

$$g = 9.81 \frac{m}{s^2} = 9.81 \cdot \frac{m}{0.3048 m} \cdot \frac{ft}{s^2} \\ = 32.174 \frac{ft}{s^2}$$

$$\rho_{H_2O} = \frac{1000 \text{ kg}}{m^3} = \frac{1000 \cdot 1 \text{ kg}}{14.59} \cdot 1 \text{ slug} \cdot \frac{0.3048^3}{m^3} \cdot \frac{m^3}{ft^3}$$

\* Dimensionless Groups:- a combination of variables in which the primary dimensions are cancel out.

$$\rightarrow \text{Mach No.} = \frac{\text{velocity}}{\text{velocity of sound}}$$

$$\rightarrow R_c = \frac{\rho V D}{\mu}$$





## \* Fluid properties:-

→ properties Related to the mass and weight.

① Mass density  $[\rho] = \text{mass/volume}$

$$[\rho] = \frac{M}{L^3}, \frac{\text{kg}}{\text{m}^3}, \frac{\text{slug}}{\text{ft}^3}, \frac{\text{lbm}}{\text{in}^3}$$

\* For water  $\rho_{\text{H}_2\text{O}} = 1000 \text{ kg/m}^3$  at  $4^\circ\text{C}$

\* For air  $\rho_{\text{air}} \approx 1.2 \text{ kg/m}^3$  at  $20^\circ\text{C}$ , 101.325 kPa.

\* table at the end of text book.

## \* Temp. Dependency

For liquids density slightly changed with increasing temp., But For gases density significantly affected by temp.

## ② Specific weight. ( $\gamma$ )

$$\gamma = \frac{\text{weight}}{\text{volume}}, \frac{\text{N}}{\text{m}^3}, \frac{\text{ML}}{\text{T}^2 \text{L}^3} = \frac{M}{\text{T}^2 \text{L}^2}$$

$$\gamma = \frac{M}{\text{T}^2 \text{L}^2}$$

$$\gamma = \frac{\text{weight}}{\text{volume}} = \frac{mg}{V} = \rho g$$

$$\gamma = \rho g$$

♥ table includes  $\gamma$  for diff fluids at diff. temp.



### ③ Specific gravity ( $S_g$ , $S_{gr}$ )

$$S_g, S_{gr} = \frac{\gamma_{\text{Fluid}}}{\gamma_{\text{water at } 40^\circ}} = \frac{\rho_{\text{Fluid}} \times g}{\rho_{\text{H}_2\text{O}} \times g}$$

$$S_g, S_{gr} = \frac{\rho_{\text{Fluid}}}{\rho_{\text{H}_2\text{O}}}$$

→ For example oil with density  $800 \text{ kg/m}^3$

$$S_{g \text{ oil}} = \frac{\rho_{\text{oil}}}{\rho_{\text{H}_2\text{O}}} = \frac{800}{1000} = 0.8$$

$$[S_g] = [-]$$

↓  
dimensionless

### ④ eqn. of state:-

For ideal gases.

$$P \cdot V = n \cdot R_u \cdot T$$

absolute Press.      total Volume      no. of moles      universal gas constant  $8314 \frac{\text{J}}{\text{kmole} \cdot \text{K}}$       total absolute temp  $[K]$

$$P \cdot V = \frac{n \cdot M}{M} \cdot R_u \cdot T$$

mass      gas const.

where  $M$  is the molecular weight

$$P \cdot V = \text{mass} \cdot R \cdot T$$

$[kg/kmol]$

$$P = \frac{\text{mass}}{V} \cdot R \cdot T$$

$$P = \rho \cdot R \cdot T$$





ex what the density of air at room temp.  $20^{\circ}\text{C}$  and standard atmospheric pressure  $101.325 \text{ kPa}$ , where  $R_{\text{air}} = 0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

$$\rho = \frac{P}{R T} = \frac{101.325 \times 10^3}{0.287 \times 10^3 \times (293.15)} \rightarrow \text{حسابه في المسألة}$$

\* Properties related to Flow of heat :-

specific heat ability of material to store thermal energy, and is defined to rise the temp. of unit mass one degree.

→ For Gases specific heat depends on the process by which the gas changes its state.

$C_v \rightarrow$  constant volume specific heat.

$C_p \rightarrow$  " " " " " " pressure " " " "

$$C_p > C_v$$

$$C_p - C_v = R$$

$$k = \frac{C_p}{C_v}, \text{ specific heat Ratio.}$$

→ Review:-

$$\rho = \frac{\text{mass}}{\text{volume}}$$

$$\gamma = \frac{\text{weight}}{\text{volume}} = \rho g$$

$$S_G = \frac{\gamma_{\text{fluid}}}{\gamma_{\text{H}_2\text{O}}} = \frac{\rho_{\text{fluid}}}{\rho_{\text{H}_2\text{O}}}$$

$$p_x = n R_u T = \rho R T$$

$$k = \frac{C_p}{C_v} \rightarrow \text{For air } k = 1.4$$





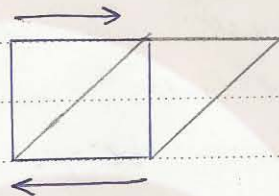
علاقة (3)

→ Viscosity اللزوجة  
{ Dynamic viscosity, absolute viscosity }

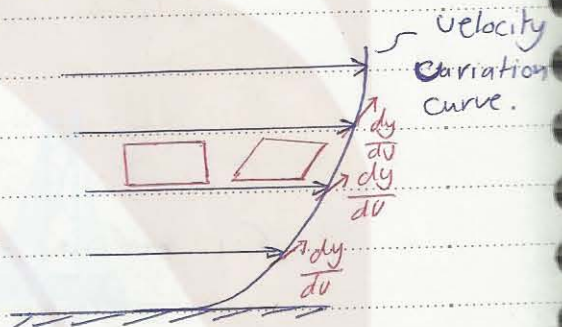
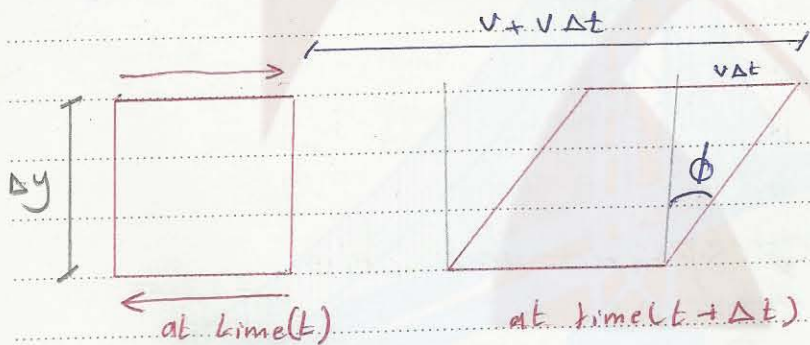
it a measure of a fluid resistance to deformation under shear stress.



♥ In solid mechanics  
 $\tau \propto$  Shear strain



♥ In Fluid mechanics  
 $\tau \propto$  Rate of strain



$$\Delta \phi = \frac{\Delta v \cdot \Delta t}{\Delta y}$$

$$\frac{\Delta \phi}{\Delta t} = \frac{\Delta v}{\Delta y}, \quad \phi', \text{ Rate of strain} = \frac{dv}{dy} = \text{velocity gradient.}$$

$$\tau \propto \frac{dv}{dy} \Rightarrow \tau = \mu \frac{dv}{dy}$$





where  $\mu$  is coefficient of viscosity {Dynamic, absolute viscosity.

$$\mu = \frac{\tau}{\left(\frac{dv}{dy}\right)} = \frac{\mu L}{\left(\frac{T^2}{L^2}\right)} = \frac{\mu}{L \cdot T^2}$$

$$\left[\frac{dv}{dy}\right] = \frac{[v]}{[y]} = \frac{\frac{L}{T}}{L} = \frac{1}{T}$$

$$[\mu] = \frac{\mu}{\frac{L \cdot T^2}{1}} = \frac{\mu}{L \cdot T} = \frac{\text{kg}}{\text{m} \cdot \text{s}} \text{ or } \text{Pa} \cdot \text{s} \text{ or } \frac{\text{N} \cdot \text{s}}{\text{m}^2}$$

→ Kinatic viscosity  $\nu = \frac{\mu}{\rho}$

\* two charts at the end of the text books as function of Temp.

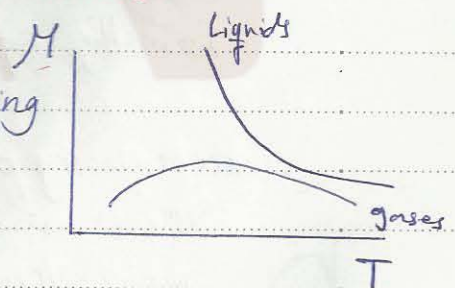
note Temp. Dependency :-

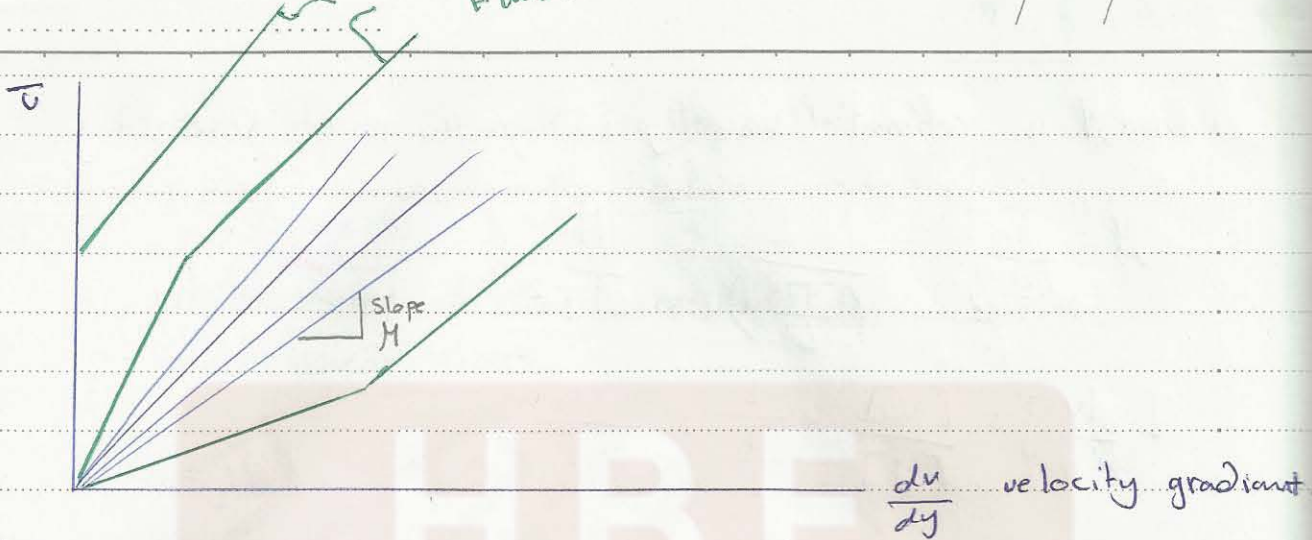
For liquid viscosity decrease with increasing temp.  
and For gases increase with increasing temp.

♡ Newtonian and non Newtonian Fluids.

Fluids that have a behavior corresponding to Newton law of viscosity called Newtonian Fluids

→ Linear relationship between  $\frac{dv}{dy}$





السرعة  
التي  
تتغير

Notes:

① the slope of variation curve must have a finite value, [impossible for the velocity variation curve to be tangent to the solid boundary]

$$\rightarrow \text{if tangent } \frac{dy}{dv} = 0 \Rightarrow \frac{dv}{dt} = \infty$$

$\tau = \infty$ , impossible

② note that the slope of velocity variation curve with distance from the solid boundary  $\frac{dv}{dy}$  decreases which mean that the  $\tau$  decreasing with distance from the solid boundary [wall]

③ the velocity of the fluids with respect to the solid surface is equal to zero  
No relationship condition





♡ Surface tension  $\sigma$  ( $\frac{N}{m}$ )

Capillary effect.

→ responsible for the dew hang drop of blood rain to take spherical shape.

→ surface tension is a function of temp. and is not effected by pressure.

→ surface tension can be changed by the addition of impurities?

T	$\sigma$
0	0.076
20	0.075
100	0.059

\* Capillary effect.

Rise or Fall of Liquid in small diameter

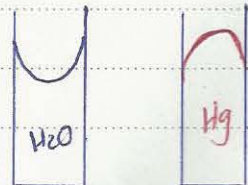


where  $\phi$  is contact or wetting angle.

$\phi < 90^\circ$  liquid wet surface.

$\phi > 90^\circ$  // not wet //

$$\phi_{Hg} = 130^\circ$$



## \* Review so

$$\tau = \mu \frac{du}{dy}$$

$$\tau \propto \frac{\Delta \phi}{\Delta t}$$

Rate of strain.

where  $\mu$ : absolute viscosity,  $\text{Pa} \cdot \text{s}$ ,  $\frac{\text{N} \cdot \text{s}}{\text{m}^2}$

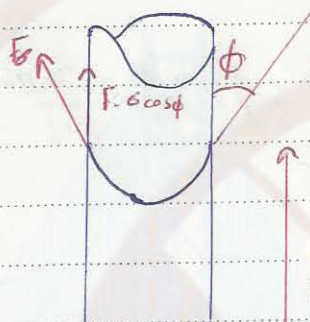
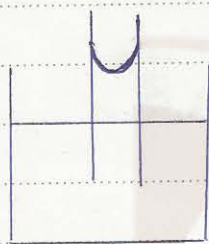
$$\text{kinematic viscosity} = \frac{\mu}{\rho}, \frac{\text{m}^2}{\text{s}}$$

Surface tension,  $\sigma$  ( $\frac{\text{N}}{\text{m}}$ )

→ Droplet

→ Bubble

→ capillary rise [cohesive (similar molecular)]  
[adhesive (different)]



contact angle.

adhesive > cohesive.

$h$  → capillary rise.

For equilibrium in y-direction:

$$\sum F_y = 0$$

$$F_s \cos \phi = mg = \rho V g$$

$$(\pi d \sigma) \cos \phi = \gamma A$$

$$\pi d \sigma \cos \phi = \frac{\pi d^2 h \gamma}{4}$$

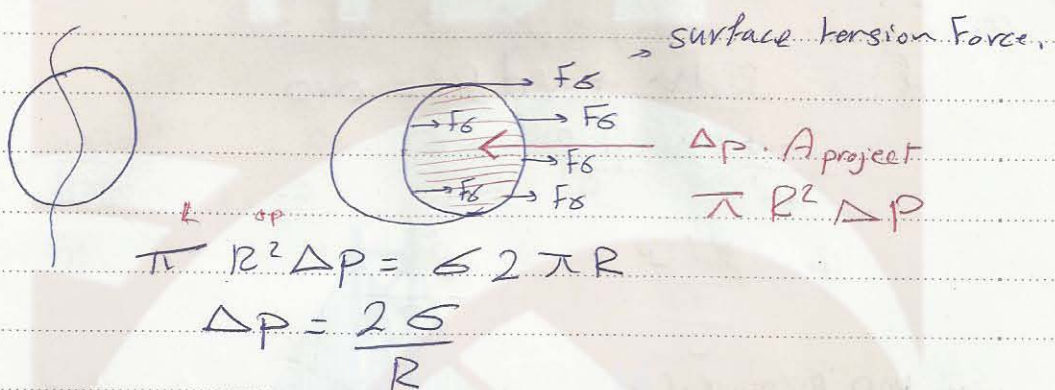
$$h = \frac{4 \sigma \cos \phi}{\gamma d}$$



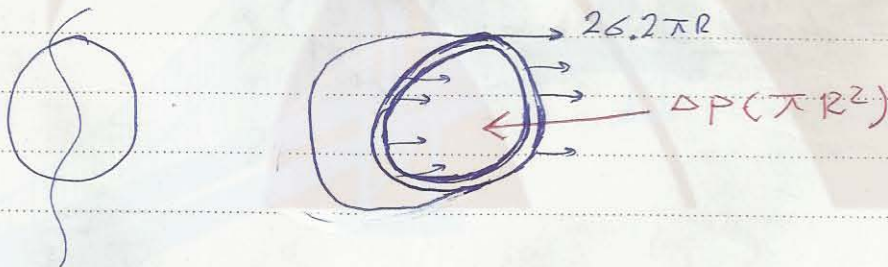
♡ Capillary rise (h) effected by :-

- 1) Surface tension  $\sigma$  (which is a function of temp).
- 2) Contact angle (rise in water fall in Hg  $\phi = 130^\circ$ )
- 3) inversely prop. to the dim.
- 4) " " " " density  $\sigma = \rho \cdot g$

\* Droplet



\* Soap Bubble.



$$4 \pi R \sigma = \pi R^2 \Delta P$$

$$\Delta P = \frac{4 \sigma}{R}$$



## \* Bulk modulus of elasticity.

$$E_v = - \frac{\Delta P}{\frac{\Delta V}{V}} \rightarrow \begin{matrix} \text{differential pressure} \\ \text{differential volume} \\ \text{total volume} \end{matrix} \quad (\text{Pa})$$

$$E_v = 2.2 \text{ GPa}$$

mass =  $\rho \cdot x$   
cost.

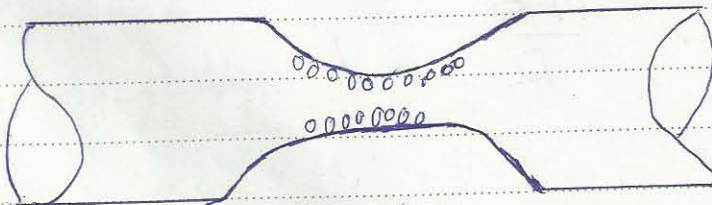
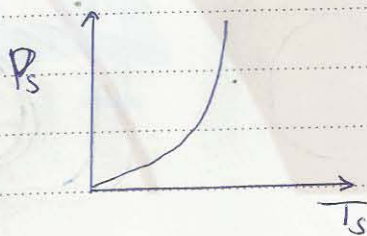
$$dH = \rho dx + v d\rho = 0$$

$$-\frac{dx}{x} = \frac{d\rho}{\rho}$$

$$E_v = \frac{dP}{\frac{d\rho}{\rho}} = \frac{\rho dP}{d\rho}$$

## \* vap pressure ( $P_v$ )

the pressure at which liquid starts to vaporize and depend on tem.



Cavitation  
كيس





5<sup>th</sup>

P. 2-6

$\rho = 31$ ,  $\gamma = 31$   $p = 300 \text{ kPa}$ , absolute,  $T = 60^\circ$

$$p = \rho R T \Rightarrow \rho = \frac{p}{R \cdot T}$$

$$R = \frac{R_u}{M}$$

$$\rho = \frac{300 \times 10^3}{518 \times (60 + 273.15)} = 1.738 \frac{\text{kg}}{\text{m}^3} = \frac{8314}{16}$$

$$= 519.62 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

$$\gamma = \rho \cdot g = (1.738)(9.81) = 17.05 \frac{\text{N}}{\text{m}^3}$$

$$R = 0.518 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

9<sup>th</sup>

P. 2-39

$$\frac{M_{310} \xrightarrow{\text{temp}}}{M_{280}}$$

O<sub>2</sub>, Does this Ratio have a value:-

(a) less than 1

✓ (b) more than 1

(c) equal 1

because :- O<sub>2</sub> gas.  $M \propto T$

$$M_{310} > M_{280}$$

9<sup>th</sup>

P. 2-34

$$u = a \left( \frac{y}{b} \right)^{1/6}$$

$$a = 10 \text{ m/s}, b = 2 \text{ mm}$$

$$\Rightarrow u = 10 \frac{y^{1/6}}{0.002^{1/6}} \Rightarrow \frac{10}{0.002^{1/6}} \cdot \frac{1}{6} y^{-5/6}$$

$$\tau = ?$$

$$\tau = \frac{\mu du}{dy} = \mu \left[ \frac{10}{6 \cdot 0.002^{1/6}} (y^{-5/6}) \right]$$

$$= 1.0 \times 10^{-3} \left[ \frac{10}{6 \cdot 0.002^{1/6}} y^{-5/6} \right]$$

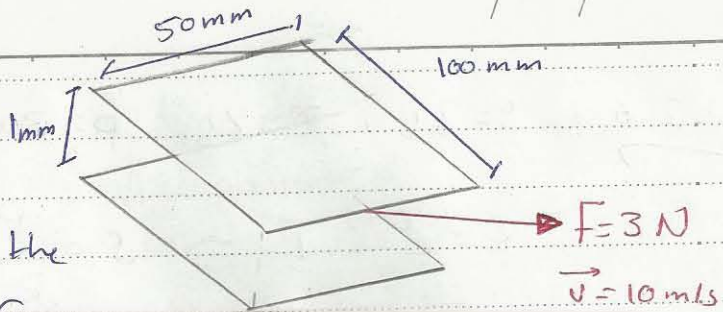
$$= 1.48483$$



qth

P. 2.33

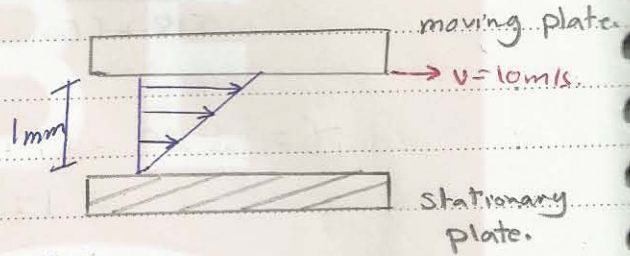
what is the viscosity of the fluid? assume that a linear distribution.



$$\tau = \mu \frac{dv}{dy}$$

For linear Relation ship.

$$\frac{dv}{dy} \approx \frac{\Delta v}{\Delta y} = \frac{10}{0.001} = 10000$$



$$\tau = F/A = \mu \frac{\Delta v}{\Delta y}$$

$$\mu = \frac{F \cdot A}{\frac{\Delta v}{\Delta y}} = \frac{(3)(0.1 \times 0.05)}{10000} = 1.5 \times 10^{-6}$$



## → Fluid static

14 / June / 2012 Thurs.

① only Fluids with velocity gradient produce shearing forces.

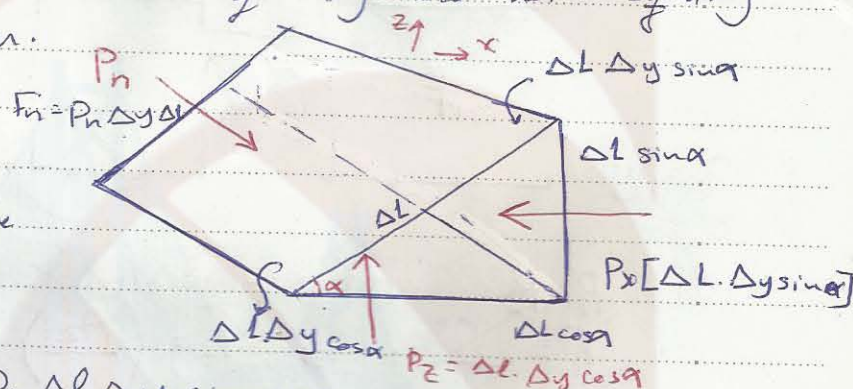
CH-3

② only Fluid with at rest only normal forces exist  
the normal force in Fluids is called pressure force

pressure :- Force per unit area.

$$p = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA} \leftarrow \text{normal component of the force.}$$

→ pressure intensity is a scalar quantity and act equally in all direction.



\* For equilibrium in the x-direction

$$\sum F_x = 0.0$$

$$P_n \Delta y \Delta L \sin \alpha - P_x \Delta L \Delta y \sin \alpha = 0.0$$

$$P_n = P_x$$

\* For equilibrium in the z-direction.

$$\sum F_z = 0.0$$

$$P_z \Delta L \Delta y \cos \alpha - P_n \Delta L \Delta y \cos \alpha - \gamma \left( \frac{1}{2} \Delta L \cos \alpha \cdot \Delta L \sin \alpha \right) \Delta y = 0.0$$

$$P_z = P_n$$

$$\begin{aligned} w &= mg \\ &= \rho \cdot V \cdot g \\ &= \gamma \cdot V \\ &= \gamma \left( \frac{1}{2} \Delta L \cos \alpha \cdot \Delta L \sin \alpha \right) \Delta y \end{aligned}$$





$$\frac{dp}{dz} = -\gamma$$

For uniform density Fluids

$\rho = \text{constant}$  throughout the fluid

$$\rho g = \gamma = \text{constant}$$

$$\frac{dp}{dz} = -\gamma \Rightarrow dp = -\gamma dz$$

$$p = -\gamma z + \text{const.}$$

$$p + \gamma z = \text{constant} \rightarrow \text{piezometric pressure.}$$

→ divided by  $\gamma$

$$\frac{p}{\gamma} + z = \text{constant} = h$$

$\frac{p}{\gamma}$  piezometric head (m)

this piezometric head is const. throughout a homogeneous.

→ if  $\gamma \neq \text{const.}$

$$\gamma = \rho g = \frac{P}{RT} g$$

$$\frac{dp}{dz} = -\frac{P}{RT} g$$

$$\frac{dp}{P} = -\frac{g}{RT} dz$$

$$\ln \frac{P_2}{P_1} = -\int \frac{g}{RT} dz$$

$$\ln \frac{P_2}{P_1} = -\frac{g}{RT_0} (z)$$



### Ch. 3 :- Fluid Statics.

→ pressure variation with elevation

$$\frac{dp}{dz} = -\gamma$$

$$\frac{dp}{dl} = -\gamma \frac{dz}{dl}$$

incompressible fluid

$$\rho = \text{const.}$$

$$\gamma = \text{const.}$$

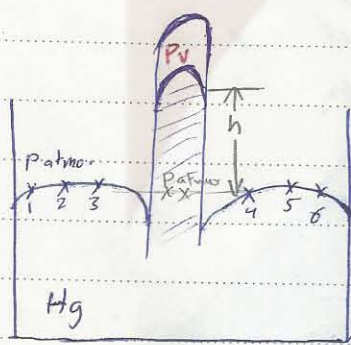
$$p + \gamma z = p = \text{const.} = \text{piezometric pressure.}$$

$$\frac{p}{\gamma} + z = \text{const.} = h = \text{piezometric head.}$$

1. pressure measurement devices :-

أجهزة القياس

→ Parameter :- devices that is used to measure the atmospheric pressure.



$$P_{atm} = P_v + \gamma_{Hg} \cdot h$$

vapor pressure

For mercury (Hg)

$$P_v = 0.000023 \text{ psi}$$

$$P_{atm} = 14.696 \text{ psi}$$

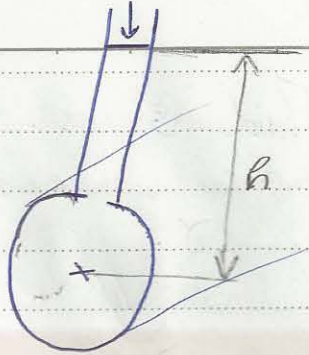
$$P_{atm} \approx \gamma_{Hg} \cdot h$$

00

vapor pressure  
ضغط البخار  
في السائل

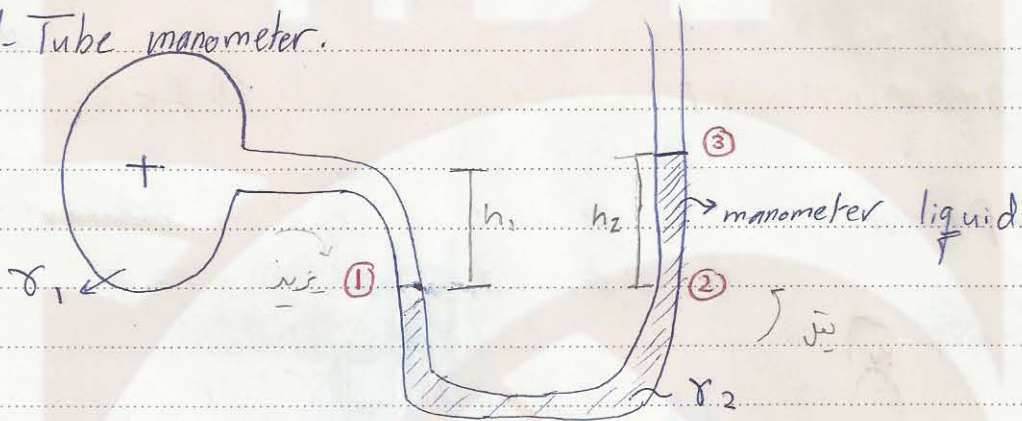
## ② Piezometer

$$P = \gamma_{\text{fluid}} \cdot h$$



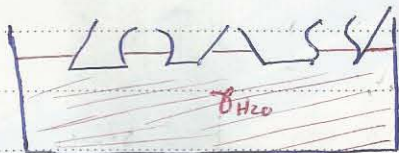
1. suitable for liquid only.
2. not suitable for high pressure.
3. " " " " subatmospheric pressure.

## ③ U-Tube manometer.



$$P_A + \gamma_1 \cdot h_1 - \gamma_2 \cdot h_2 = 0$$

$$P_A = \gamma_2 \cdot h_2 - \gamma_1 \cdot h_1$$

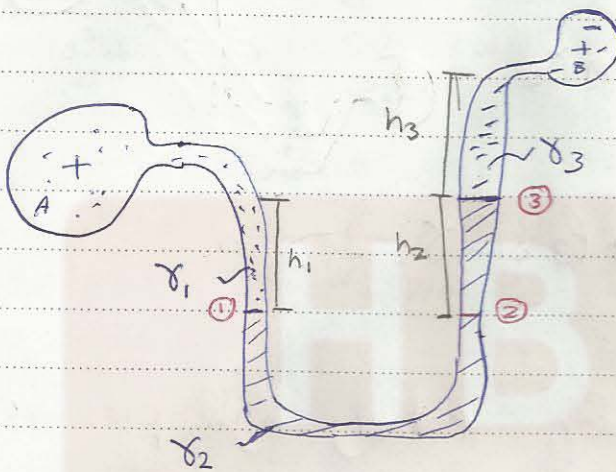


→ here the pressure the same at all point and only depends on depth.

هنا الضغط متساوي في كل نقطة ويعتمد فقط على العمق.



## Differential U-tube manometer.



$$P_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = P_B \quad (P_A - P_B)$$

$$P_A - P_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

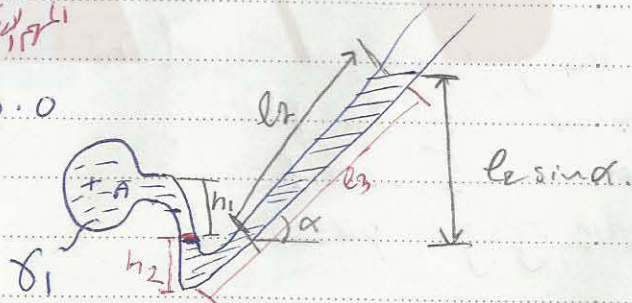
$$P_B + \gamma_3 h_3 + \gamma_2 h_2 + \gamma_1 h_1 = P_A$$

$$P_A - P_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

⇒ Inclined manometer

$$P_A + \gamma_1 h_1 - l_2 \sin \alpha \gamma_2 = 0.0$$

$$P_A = l_2 \sin \alpha \gamma_2 - \gamma_1 h_1$$



$$P_B + \gamma_1 h_1 + \gamma_2 h_2 - l_2 \sin \alpha \gamma_2 = 0.0$$

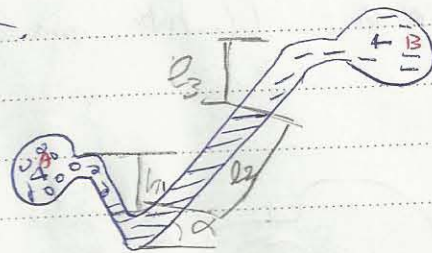
$$P_B = l_2 \sin \alpha \gamma_2 - \gamma_2 h_2 - \gamma_1 h_1$$

$$= (l_2 \sin \alpha - h_2) \gamma_2 - \gamma_1 h_1$$



⇒ Diff. Inclined manometer.

$$P_A + \gamma_1 h_1 - \gamma_2 l_2 \sin \theta_2 - \gamma_3 h_3 = P_B$$



$$P_A - P_B = \gamma_3 h_3 + l_2 \sin \theta_2 - \gamma_1 h_1$$

→ manometer

① is not suitable for relatively high pressure.

② is not suitable when there is variation in pressure.

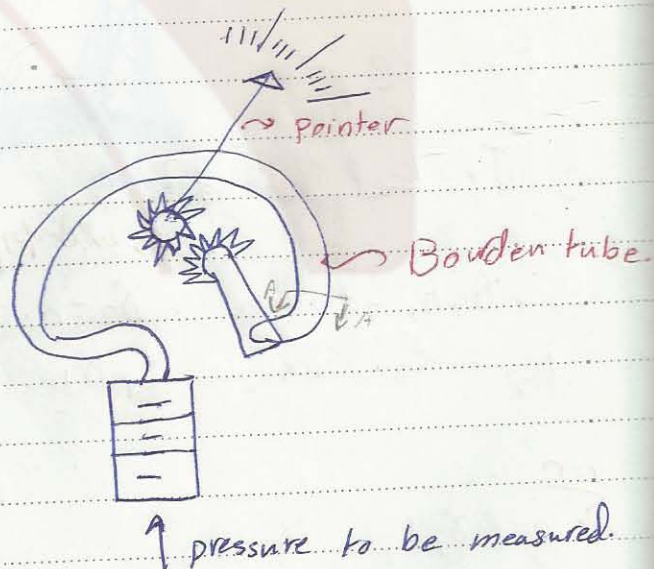
③ Time consuming [not difficult to measure]

♡ pressure Transducers.

♡ Bourden gage

\* this gage can be used to measure subatmospheric pressure [vacuum] and tue gage press.

$$[P > P_{atm}]$$





3/15/12



18/June/2012 mon

P.3-11 S.G. oil = ?

$$P_A + \gamma_{oil} \times 1 = P_B$$

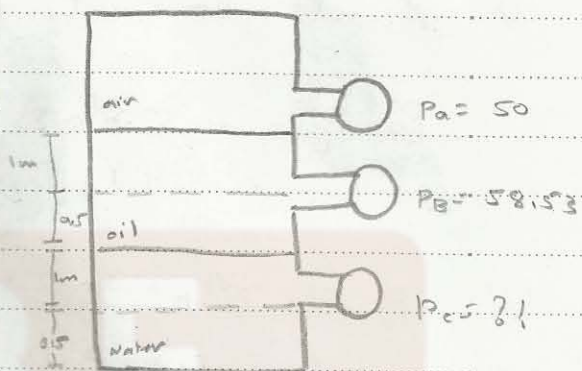
$$50 \times 10^3 + \gamma_{oil} \times 1 = 58.53 \times 10^3$$

$$\gamma_{oil} = 8.53 \text{ kN/m}^3$$

$$P_B + 0.5 \times \gamma_{oil} + \gamma_{water} \times 1 = P_C$$

$$58.53 \times 10^3 + 0.5 \times 8.53 \times 10^3 + 9810 = P_C$$

$$P_C = 72.6 \text{ kPa}$$

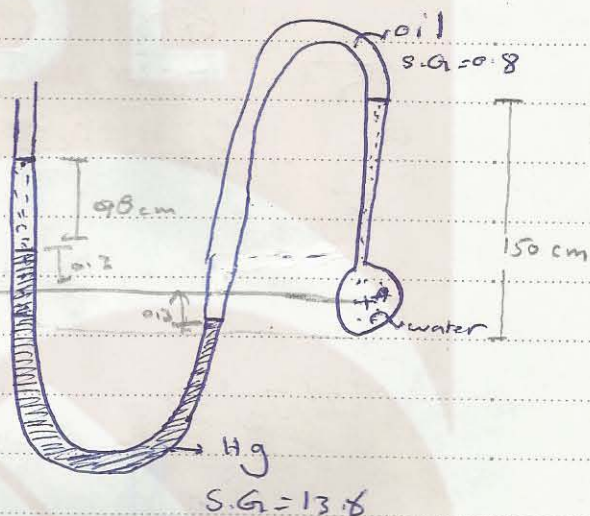


P.3-39

$$P_A = 1.5(9810) + 1.8(0.8 \times 9810)$$

$$- (13.6(9810) \times 0.6) - 0.9(9810) = 0$$

$$P_A = 89.5 \text{ kPa}$$



eg. ① Determine the diff. in pressure.

$$P_A - P_B$$

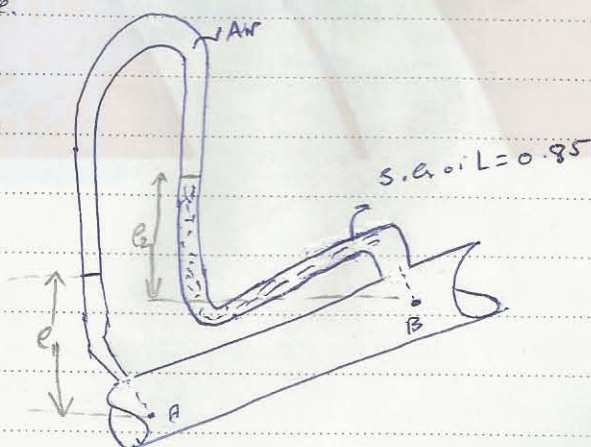
② the diff. in piezometric head.

$$Z_A = 10 \text{ m}$$

$$Z_B = 11 \text{ m}$$

$$l_1 = 1 \text{ m}$$

$$l_2 = 0.5 \text{ m}$$





$$\textcircled{1} \quad P_B - l_2 \cdot \gamma_{oil} + l_1 \cdot \gamma_{oil} = P_A$$

$$\begin{aligned} P_A - P_B &= -0.5 \times 0.85(9810) + 1 \times 0.85 \times 9810 \\ &= 4.17 \text{ kPa.} \end{aligned}$$

$$\textcircled{2} \quad P_B - l_2 \cdot \gamma + l_1 \cdot \gamma = P_A$$

$$l_1 = z_B - z_A$$

$$\frac{P_B}{\gamma} - \frac{l_2 \cdot \gamma}{\gamma} + \frac{(z_B - z_A) \cdot \gamma}{\gamma} = \frac{P_A}{\gamma}$$

$$\frac{P_A}{\gamma} + z_B - l_2 - z_A = \frac{P_A}{\gamma}$$

$$\frac{P_A}{\gamma} + z_A - \left( \frac{P_B}{\gamma} + z_B \right) = -l_2$$

$$\begin{aligned} h_A - h_B &= -l_2 \\ &= -0.15 \text{ m.} \end{aligned}$$

piezometric head

$$p + \gamma z = \text{const.}$$

$$\frac{p}{\gamma} + z = h = \text{piez. head}$$

$$h_A - h_B$$

50





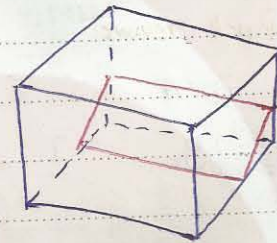
## ♡ Hydrostatic force on plane surface.

→ Horizontal surface and surface that exposed to gas pressure essentially have anniform pressure distribution on the entire surface area.

Force = uniform pressure  $\times$  area.

→ if a plane surface not horizontal and if it is acted on by hydrostatic force such as that produce by static fluid. then the pressure is linearly dist. over the surface.

$$\begin{aligned} dF &= P \cdot dA \\ &= \gamma h \cdot dA \\ &= \gamma y \cdot \sin \alpha \cdot dA \end{aligned}$$

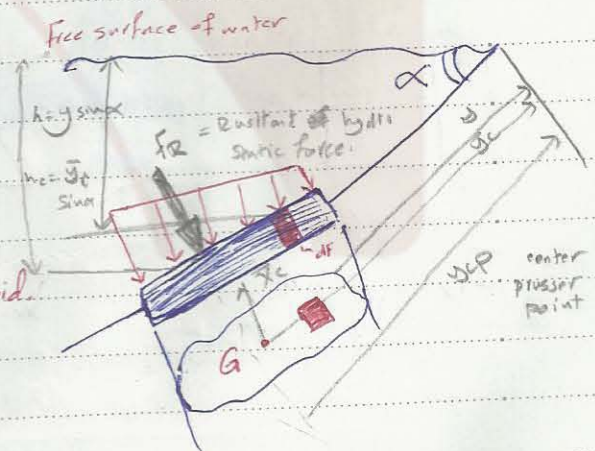


$$\begin{aligned} F_R &= \int dF = \int \gamma y \sin \alpha \cdot dA \\ &= \gamma \sin \alpha \int y \cdot dA \end{aligned}$$

First moment of area

$$\int y \cdot dA = \bar{y}_c \cdot A$$

$\bar{y}_c$  : y coordinate of the centroid.



$$F_R = \gamma \sin \alpha \bar{y}_c \cdot A$$

$$= \gamma \cdot h_c \cdot A = P_c \cdot A$$

The last eqn. states that the hydrostatic force acting on an inclined plain is equal to the hydrostatic pressure at the Centroid multiplied by the entire area.





moment about  $O_c$ :-

$$F_R \cdot y_{cp} = \int dF \cdot y$$

$$\sigma \bar{y}_c \sin \alpha \cdot A \cdot y_{cp} = \int \sigma \sin \alpha \cdot y \cdot dA \cdot y$$

$$\sigma \bar{y}_c \sin \alpha \cdot A \cdot y_{cp} = \sigma \sin \alpha \int y^2 \cdot dA$$

$$y_{cp} [\bar{y}_c \cdot A] = \int y^2 \cdot dA$$

→ second moment of area  
moment of inertia. ( $I_o$ )

$$I_o = \bar{I}_c + \bar{y}_c^2 A \quad \text{parallel axis theorem.}$$

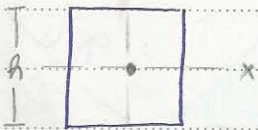
$$y_{cp} = \bar{y}_c + \frac{I}{\bar{y}_c \cdot A}$$

[A] Q's

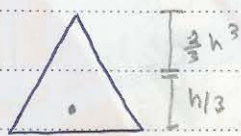
Quick review.

$$F_R = P_c \cdot A = \sigma \cdot \bar{h}_c \cdot A = \sigma \cdot \bar{y}_c \cdot \sin \alpha \cdot A$$

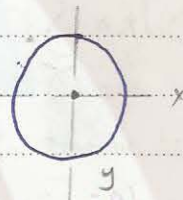
$$y_{cp} = \bar{y}_c + \frac{I}{\bar{y}_c \cdot A} \quad \rightarrow \text{2nd moment of area [moment of inertia]}$$



$$\bar{I}_{xx} = \frac{1}{12} b h^3$$



$$\bar{I}_{xx} = \frac{1}{36} b \cdot h^3$$



$$\bar{I}_{xx} = \bar{I}_{yy} = \frac{\pi}{4} R^4$$





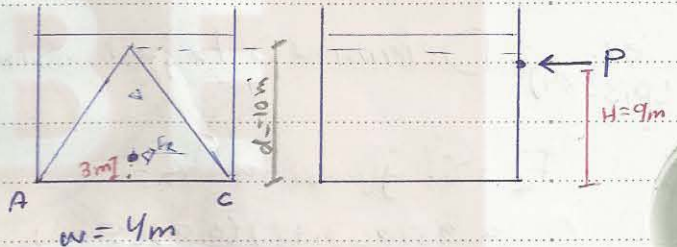
ex. The triangular gate ABC is pivoted at the bottom edge AC and closes a triangular opening ABC in the wall of the tank

$w = 4\text{m}$ ,  $H = 9\text{m}$ ,  $d = 10\text{m}$  -

1) determine the hydrostatic  $F_R$ .

2) " " Force  $P$  required to hold gate closed.

$$\begin{aligned} F_R &= \bar{P}_c \cdot A \\ &= \gamma_{H_2O} \cdot \bar{y}_c \cdot A \\ &= 9810 (10 - 3) \frac{1}{2} (4 \times 9) \\ &= 1.236 \text{ MN} \end{aligned}$$



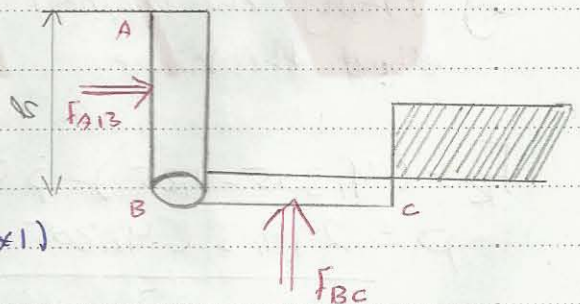
$$y_{cp} = \bar{y}_c + \frac{I}{\bar{y}_c \cdot A} = [10 - 3] + \frac{\frac{1}{36} (4)(9)^3}{7(\frac{1}{2})(4)(9)} = 7.643 \text{ m}$$

By taking the moment about point A:-

$$F_R [10 - 7] + P(9) = 0 \Rightarrow P = 323.69 \text{ kN}$$

ex. the minimum water level in gate surface AB is 1 ft at what depth  $h$  will the gate automatically open? neglect the weight of gate, both AB, BC, point as unit about the hinge B.

$$\begin{aligned} F_{AB} &= \gamma \cdot \bar{h}_c \cdot A \\ &= \gamma \cdot \left(\frac{h}{2}\right) h(1) \quad \rightarrow \text{unit depth} \\ &= \frac{1}{2} \gamma h^2 \end{aligned}$$



$$\begin{aligned} F_{BC} &= \gamma \cdot \bar{h}_c \cdot A_{BC} = \gamma (h) (4 \times 1) \\ &= 4 \gamma \cdot h \end{aligned}$$

$$\begin{aligned} y_{cp_{AB}} &= \bar{y}_c + \frac{I}{\bar{y}_c \cdot A_{AB}} = \frac{h}{2} + \frac{\frac{1}{12} (1)(h^3)}{\frac{h}{2} (h)(1)} \\ &= \frac{h}{2} + \frac{h}{6} = \frac{4h}{6} = 0.667 h \end{aligned}$$





$$\sum M_B = 0.0$$

$$-F_{AB} [h - 0.1667h] + F_{BC} [2] = 0.0$$

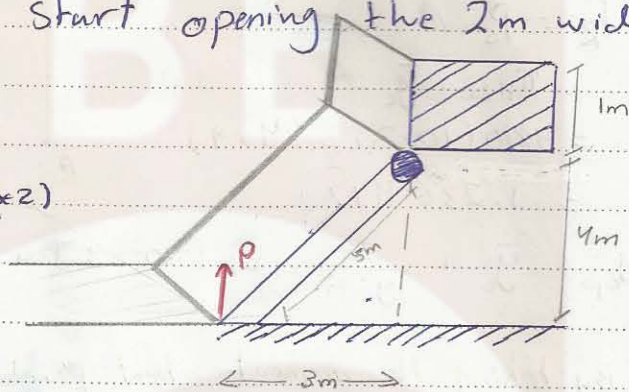
$$-\frac{1}{2} \cdot 8 h^2 (0.1667h) + 8 h = 0.0$$

$$0.1667 h^3 - 8 h = 0.0$$

$$\text{So, } h = 0.0 \text{ or } h = \sqrt[3]{\frac{8}{0.1667}} = 6.93 \text{ ft}$$

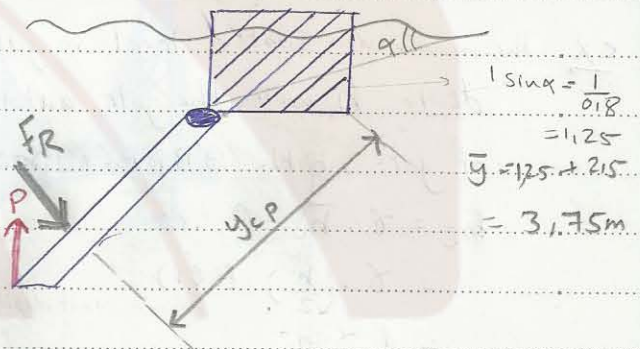
ex. (p.3-84) Determine  $p$  to just start opening the 2m wide gate.

$$\begin{aligned} F_R &= \gamma \cdot \bar{y}_c \cdot \sin \alpha \cdot A \\ &= 9810 (3.75) (0.8) (5 \times 2) \\ &= 294,3 \text{ kN} \end{aligned}$$



$$\begin{aligned} y_{cp} &= \bar{y}_c + \frac{I}{\bar{y}_c \cdot A} \\ &= 3.75 + \frac{\frac{1}{12} (2) (5^3)}{(3.75) (2) (5)} \\ &= 4.305 \text{ m} \end{aligned}$$

By taking the moment about the hinge.



$$F_R (4.305 - 1.25) - p (3) = 0.0$$

$$p = \frac{294,3 (4.305 - 1.25)}{3} = 299.7 \text{ kN}$$





[9] Jrids

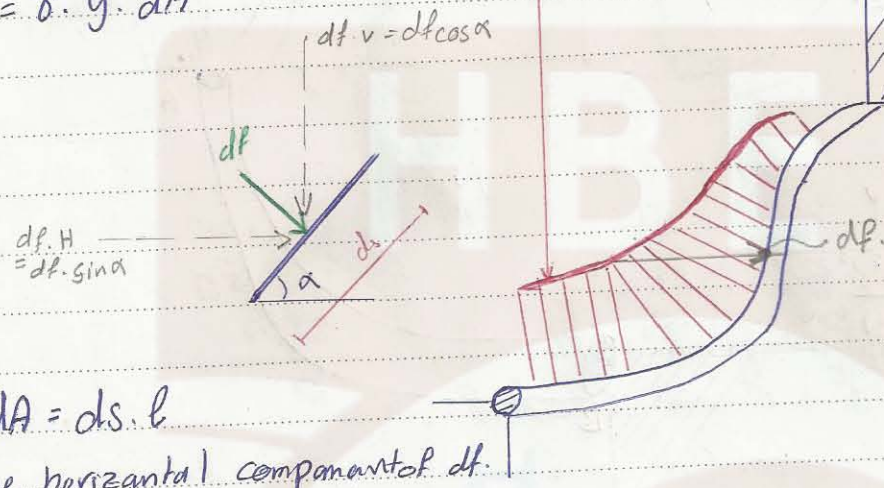
# Hydrostatic force on curved surface.

depth of gate (e)

differential force  $df$  :-

$$df = P \cdot dA$$

$$= \gamma \cdot y \cdot dA$$



$$dA = ds \cdot l$$

The horizontal component of  $df$ .

$$df_H = df \sin \alpha = \gamma \cdot y \cdot l \cdot ds \sin \alpha$$

$$= [\gamma \cdot y \cdot dA_v]$$

$$\sum df_H = F_H = \sum \gamma \cdot y \cdot dA_v$$

$$= \gamma \sum y \cdot dA_v = \gamma \cdot \bar{y}_v \cdot A_v$$

This eqn. state that the horizontal component of the hydrostatic force acting on a curved surface is equal to the pressure at the centroid of the vertical projection Area.

$$y_{cp} = \bar{y}_v + \frac{I}{\bar{y}_v \cdot A_v}$$

the second moment of area for the vertical projection area.

the y coordinate of the centroid of the vertical projection area.

$$df_v = df \cdot \cos \alpha$$

$$= \gamma \cdot y \cdot l \cdot ds \cos \alpha = \gamma \cdot dA_v$$

the volume of the liquid above the curve





ex. determine the component of the hydrostatic force acting on the gate AB.

$$F_H = \gamma \cdot \bar{y}_v \cdot A_v$$

$$= 9810 \left( 4 + \frac{2}{2} \right) (2)$$

$$= 98100 \text{ N}$$

$$y_{cp} = \bar{y}_v + \frac{I}{\bar{y}_v \cdot A_v}$$

$$= 5 + \frac{\frac{1}{12} (1) (2)^3}{5 \times (2) (1)}$$

نقطة المركز الثقل = 5.0667 m

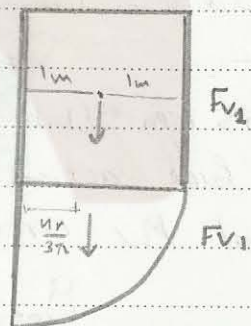
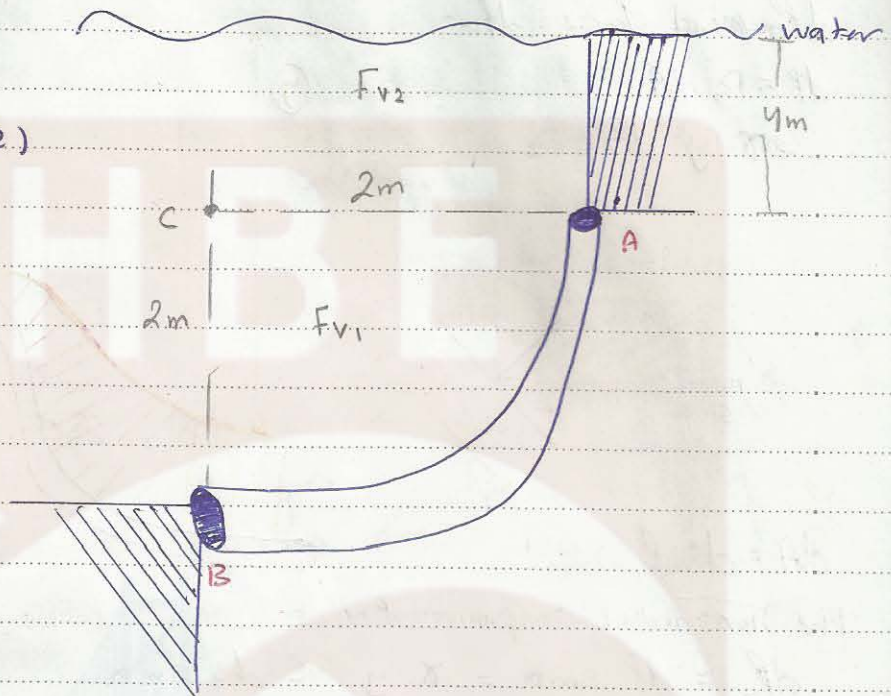
$$F_v = F_{v1} + F_{v2}$$

$$= \frac{\pi}{4} R^2 (1)(8) + (2)(4)(1)(8)$$

$$= \frac{\pi}{4} (4)(9810) + 8(9810) = 109299 \text{ N}$$

$$109299 x_{cp} = 78480 (1) + \frac{30819 (4 \times 2)}{3\pi}$$

$$x_{cp} = 0.96$$









**Buoyant Force:** which is equal to the weight of the displaced volume and line of action in the upward direction passes the centroid of the displaced volume.

→ Buoyant Force: is independent of the depth below the free surface of the density.

→ no depend on the density of the immersed body.

### \* Archimedes principle

The buoyant force acting on an immersed body is equal to the weight of displaced liquid, and it acts in the volume in the upward direction.



Floating  $\rho_b < \rho_f$



Suspended  $\rho_b = \rho_f$

→ Buoyant  $F_B$  is a function of the fluid density ( $\rho$ )  
Therefore the buoyant force is expected to be small  
here for it may be neglected.

### → Hydrometer:-

a simple device that may be used to measure specific gravity of fluid.

$$S.G = \frac{\rho_{\text{fluid}}}{\rho_{H_2O}} = \frac{\rho_{f1} \cdot g}{\rho_{H_2O} \cdot g} = \frac{\rho_{f1}}{\rho_{H_2O}}$$



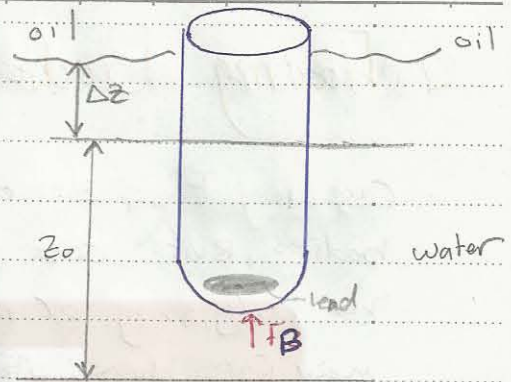
$$\begin{aligned} W_{hydro} &= F_B = \rho_{H_2O} \cdot V \\ &= \rho_{H_2O} \cdot A \cdot Z_0 \\ &= \rho_{H_2O} \cdot A \cdot Z_0 \end{aligned}$$

~ For oil  $\Rightarrow$

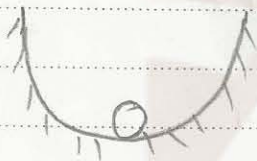
$$\begin{aligned} W_{oil} &= \rho_{oil} \cdot A \cdot (Z_0 + \Delta Z) \\ &= W_{hydro} \end{aligned}$$

$$\rho_{H_2O} \cdot A \cdot Z_0 = \rho_{oil} \cdot A \cdot (Z_0 + \Delta Z)$$

$$S.G. = \frac{\rho_{fluid}}{\rho_{H_2O}} = \frac{Z_0}{Z_0 + \Delta Z}$$



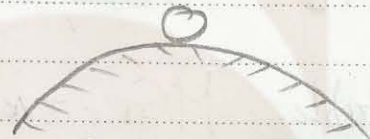
\* Stability of immersed and floating bodies.



Stable

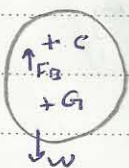


neutral  
stable



unstable.

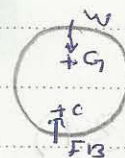
→ The stability of immersed and floating body depends on the position of  $C$  [center of the Buoyancy] and  $G$  [center of gravity].



Stable.. heavy bottom body  
[submarine]



neutral

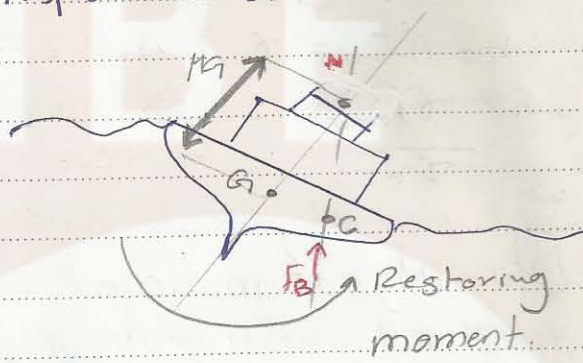
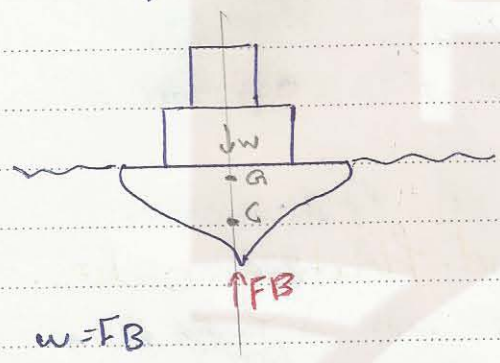


unstable.



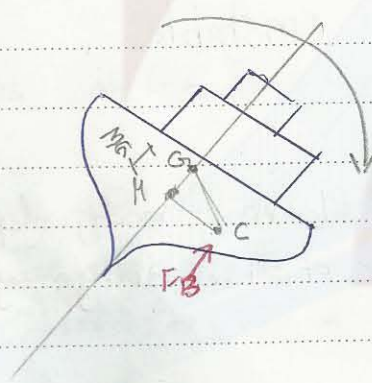
## ♥ Floating Bodies:-

Case 1, case 2 are identical to that for immersed bodies, But case 3 For floating body with  $G$  above  $C$ .  
The body may or may be not stable depending on the new position of  $C$  respect to  $G$ .



$M$  is meta center, point of intersection of the line of action bought. Force before and after rotate.

Stable.  $MG$  : +ve  $\rightarrow M$  located above  $G$ .



over turning moment  
unstable,  $MG$  : -ve,  $M$  below  $G$ .

$$MG = \frac{I}{h} - CG$$

$h$  is Distance between center of body and center of gravity.

p. (3-134)



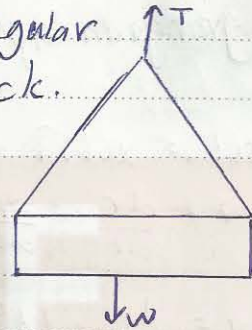


قائمة الامتحان

ex. a crane is used to lower weight into the sea  $[p = 125 \frac{\text{kg}}{\text{m}^3}]$   
for an underwater construction  $\rightarrow$  determine the tension in  
the rope of the crane due to a rectangular  
(0.4 x 0.4 x 3) m  $\rightarrow$  concrete block.

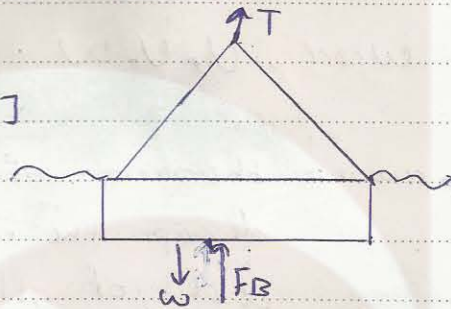
$\Sigma F = \text{zero}$   $\rightarrow$  متوازن

$$\begin{aligned} T &= w = mg \\ &= 0.48 \times 9.81 \times 23000 \\ &= 10.8 \text{ kN} \end{aligned}$$



$$\Sigma F = T + F_B - w = 0$$

$$\begin{aligned} T &= w - F_B = 10.8 - [8_{\text{wake}} \times 0.48] \\ &= 5.9 \end{aligned}$$



p (3,13).

# CH(4) Fluid in motion.

→ velocity, acceleration

CH.4  
Lagrangian

→ Lagrangian approach.

$$r(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

$$v = \frac{dr}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$v = \frac{dr}{dt} = u\hat{i} + v\hat{j} + w\hat{k}$$

where,  $u, v, w$  component of the velocity.

→ remark:- the motion of one molecule is not adequate to describe the flow, you have to find the velocity of all molecules, and to solve equation of motion ( $\sum F = ma$ )

→ Eulerian approach:-

$$u = f_1(x, y, z, t)$$

$$v = f_2(x, y, z, t)$$

$$w = f_3(x, y, z, t)$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$a_x = \frac{du}{dt}$$

$$a_x = \frac{dx}{dt} \frac{\partial u}{\partial x} + \frac{dy}{dt} \frac{\partial u}{\partial y} + \frac{dz}{dt} \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

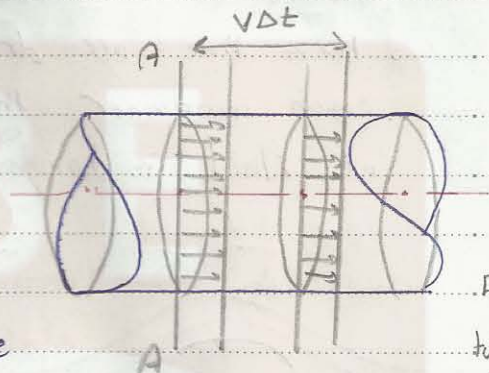


## \* Rate of flow :-

the volume of fluid  $\Delta V$  that pressure section  $\Delta A$  during time interval.

$$\Delta V = (V \Delta t) A$$

Cross section area  
of the fluid passage.



For  
turbulent  
flow.

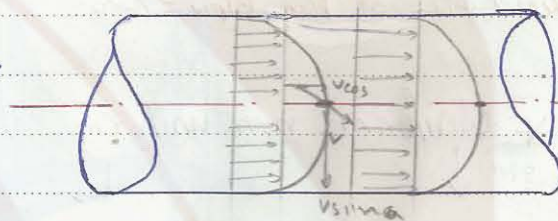
→ rate of flow volumetric flow rate

$$\text{Discharge } Q = \frac{\Delta V}{\Delta t} = \frac{V \Delta t A}{\Delta t} = VA$$

$$Q = \frac{L}{T} \times L^2 = \frac{L^3}{T} \quad \left( \frac{m^3}{s}, \frac{ft^3}{s}, \frac{L^3}{s} \right)$$

in most cases velocity varies across the passage.

$$Q = \int_S v \, dA$$



For  
laminar  
flow

\* acceleration :- the rate of change in fluid particles with respect to time.

In General :-

$$Q = \int_S \bar{v} \cdot dA$$

$$\bar{v} \cdot A = \int_S v \cdot dA$$

↳ average or mean velocity.

$$\bar{v} = \frac{Q}{A} = \frac{\int_S v \cdot dA}{A}$$

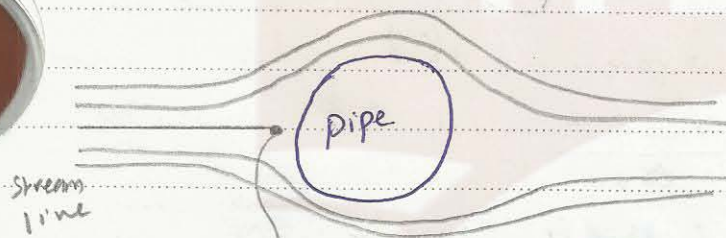




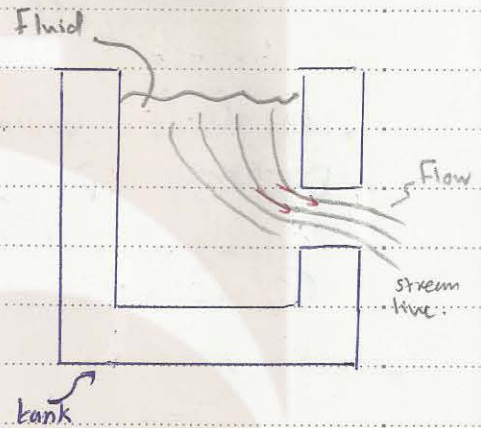
Stream line and flow pattern:- عبارة عن خطوط موجودة في مجال التدفق توضح السرعة والاتجاه للتدفق.

→ Lines in flow field to give information about speed and direction of the flow.

→ These lines are drawn in the flow field in such a manner that velocity of each molecule of fluid is tangent to the stream line of this point.



- this point called stagnation point.
- deceleration of the flow = zero.



\* Uniform, non uniform flow:-

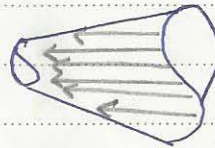
no convective acceleration

↓  
the velocity doesn't change from one point to another

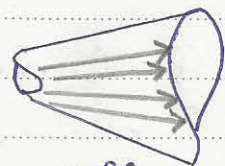
$$\frac{dv}{ds} = 0$$

→ velocity change with space.

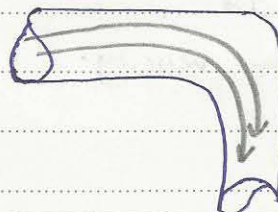
$$\frac{dv}{ds} \neq 0$$



nozzle



diffuser







## \* Steady and un Steady flow:-

no local acceleration

- steady:- velocity doesn't change with respect to time.  $\frac{dv}{dt} = 0$
- non steady:-  $\frac{dv}{dt} \neq 0$

## \* Laminar and Turbulent flow:-

- turbulent:- mixing effect, eddy current and dominants.
- laminar:- " " is negligible.

$$Re = \frac{\rho \cdot v \cdot d}{\mu}$$

Coefficient to distinguish the fluid flow.

- $\rho$ :- density
- $v$ :- velocity
- $d$ :- diameter
- $\mu$ :- dynamic viscosity

- $Re < 2000 \rightarrow$  laminar flow
- $Re > 2000 \rightarrow$  turbulent flow.

## \* Flow with respect to dimension:-

- one dimensional flow- (flow in pipe)
- Two " " ( " between two plate)
- Three " "

## \* Flow with respect to boundary:-

- Bounded flow.
- Semi-Bounded flow
- Unbounded flow.



## \* Tangential and normal Component of acceleration \*

$$v = v(s, t)$$

$$a_t = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} + \frac{dv}{dt} \cdot \frac{dt}{dt}$$

So

$$a_t = \frac{\partial v}{\partial s} \cdot \frac{\partial s}{\partial t} + \frac{\partial v}{\partial t}$$

$$a_n = \frac{v^2}{r}$$

r :- radius of curvature

Review for Euler's approach.

$$U = F(x, y, z, t)$$

$$a_x = U \frac{du}{dx} + V \frac{du}{dy} + W \frac{du}{dz} + \frac{du}{dt}$$

$$a_y = U \frac{dv}{dx} + V \frac{dv}{dy} + W \frac{dv}{dz} + \frac{dv}{dt}$$

$$a_z = U \frac{dw}{dx} + V \frac{dw}{dy} + W \frac{dw}{dz} + \frac{dw}{dt}$$

Convective  $\bar{a}$   
with respect to space.

local  $\bar{a}$   
with respect  
to time



## \* Basic control Volume approach.

control volume maybe defined as a region in space (containing quantity of matter) to establish to aid in solving flow problems.

The region is surrounded by surface which is called control surface, this surface separates the control from surrounding.

(B) → Extensive property → depend on mass:  $M, E, P$

(B) → Intensive property → independent of mass:  $T, P, V, \rho$

\* consider the control volume

$$\begin{aligned} Q_1 &= V_1 \cdot A_1 = V_1 \cdot A_1 \cos 0 \\ &= V_1 \cdot A_1 \cos 180 \end{aligned}$$

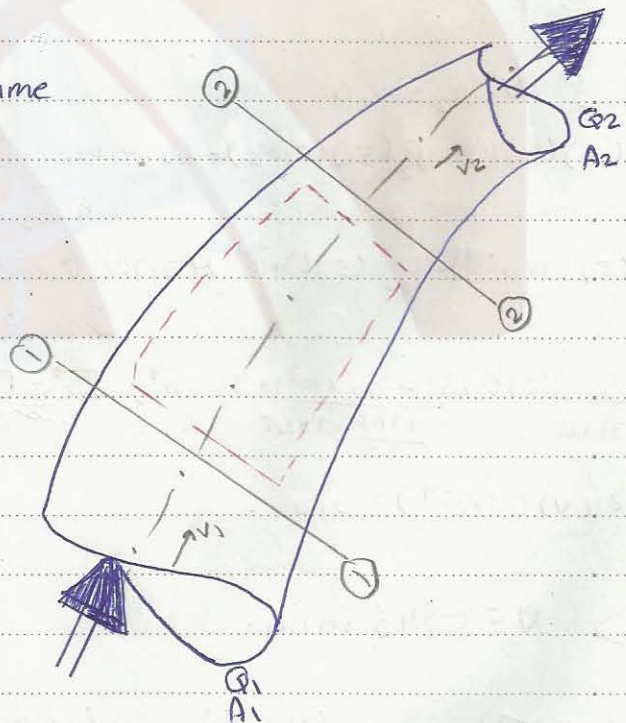
$$Q_1 = -V_1 \cdot A_1$$

$$Q_2 = V_2 \cdot A_2 = V_2 \cdot A_2 \cos 0$$

$$Q_2 = V_2 \cdot A_2$$

Flow out - Flow in

$$V_2 \cdot A_2 - V_1 \cdot A_1 = \sum V \cdot A$$







ex. the velocity field for fluid is given by

$$v = \underbrace{2x^2t}_{u} \hat{i} + \underbrace{3xy^2}_{v} \hat{j} + \underbrace{2xz}_{w} \hat{k}$$

Find  $a_x$  at  $(1, 2, 2)$  when  $t = 1$  sec.

الحل:

$$a_x = (2x^2)(4x)t^3 + 0(3xy^2) + 0(2xz) + (2x^2) \\ = 10 \text{ m/s}^2$$

المطلوب:

$$a_y = 6x^2y^2t + 18(x^2)(y^3)$$

المطلوب:

$$a_z = 2x^2t(2z) + 0 + (2xz)t(2x) + 0$$

P(4, 78) ex. Determine the exit velocity.

$\sum \dot{P} \cdot V \cdot A_2 = 0.10$

$\frac{P}{RT} = P_1(5)(901) + P_2(5)(10103) = 2.2 \cdot 0.103$

$-P_{CH_4}(5)(1 \times 10^{-4}) - P_{O_2}(5)(3 \times 10^{-4}) + 2.2(5)(3 \times 10^{-4}) = 0$

$\frac{-200 \times 10^3}{8314} \times 37315 + \frac{200 \times 10^3}{32} \times 37315 + 2.2(5)(3 \times 10^{-4}) = 0.10$

$\Rightarrow V = 5.46 \text{ m/sec}$

Diagram showing a control volume with two inlet streams (CH<sub>4</sub> and O<sub>2</sub>) and one outlet stream (mixture). The inlet velocities are  $V_1 = 5 \text{ m/s}$  and  $V_2 = 5 \text{ m/s}$ . The outlet velocity is  $V$ . The inlet areas are  $A_1 = 3 \text{ cm}^2$  and  $A_2 = 3 \text{ cm}^2$ . The outlet area is  $A = 3 \text{ cm}^2$ . The inlet pressures are  $P_1 = 2.2 \text{ kg/m}^3$  and  $P_2 = 2.2 \text{ kg/m}^3$ . The outlet pressure is  $P = ?$ .



## Reynold's Transport theorem.

$$\left. \frac{dB}{dt} \right|_{\text{sys}} = \frac{d}{dt} \int_{\text{c.v}} \rho \cdot B \cdot dV + \int_{\text{c.s}} \rho B \vec{v} \cdot d\vec{A}$$

→ Continuity equation

$$B = \text{mass} = m, \quad \beta = \frac{B}{m} = \frac{m}{m} = 1$$

$$\left. \frac{dm}{dt} \right|_{\text{sys}} = \frac{d}{dt} \int_{\text{c.v}} \rho(1) \cdot dV + \int_{\text{c.s}} \rho(1) \vec{v} \cdot d\vec{A}$$

by definition the amount of matter in the system is cons.

$$\left. \frac{dm}{dt} \right|_{\text{sys}} = \text{Zero.}$$

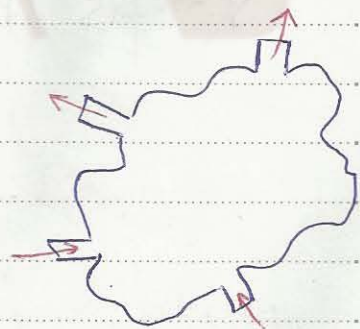
$$\int_{\text{c.s}} \vec{v} \cdot d\vec{A} = - \frac{d}{dt} \int_{\text{c.v}} \rho \cdot dV \Rightarrow \text{integral form of continuity eqn.}$$

→ For steady flow.

$$\int_{\text{c.s}} \rho \cdot \vec{v} \cdot d\vec{A} = 0 \quad \text{مجموع}$$

→ For uniform velocity

$$\sum \rho \cdot \vec{v} \cdot A = 0, 0 \quad \text{مجموع}$$





Q10] Q13



\* For one-dimensional fluid:-

steadily

$\rho \Rightarrow \text{const.}$

incompressible

$$\sum P \cdot V \cdot A = 0.0$$

$$\rho_1 = \rho_2 = \text{const.}$$

$$\rho_1 \cdot V_1 \cdot A_1 = \rho_2 \cdot V_2 \cdot A_2$$

$$V_1 \cdot A_1 = V_2 \cdot A_2$$

$$Q_1 = Q_2$$

$$p(4.78)$$

$$p(4.82)$$

$$p(4.85)$$

$$ex. (4.8)$$

→ Review:-

Continuity equation in the differential form

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0.0$$

where  $(u, v, w)$  the component of the velocity in  $(x, y, z)$  respectively.

$\rho$ :- is the density of fluid.



$$\vec{\nabla} = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \rightarrow \text{gradient}$$

$$\frac{Dz}{Dt} = \frac{u}{\partial x} \frac{\partial H}{\partial x} + v \frac{\partial H}{\partial y} + w \frac{\partial H}{\partial z} + \frac{\partial H}{\partial t} \rightarrow \text{substantive derivation}$$

$$\rho [\vec{\nabla} \cdot \vec{v}] + \frac{D\rho}{Dt} = 0.0$$

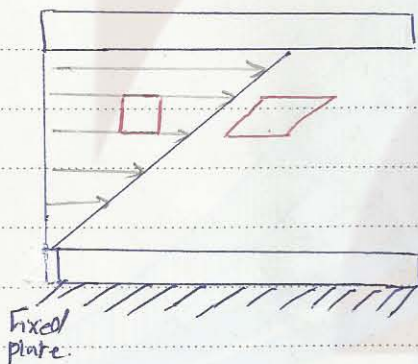
For steady state incompressible fluid.

دائرة  
التي

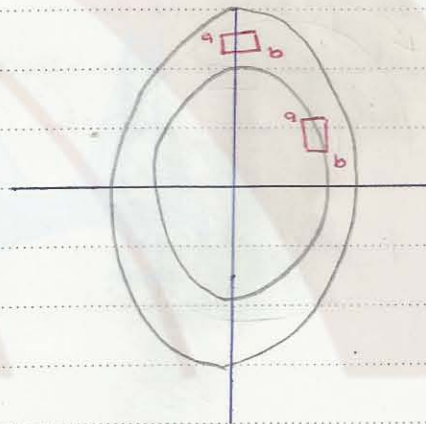
$$\rho = \text{const} \Rightarrow \frac{\partial}{\partial t} (\text{any thing}) = \text{zero}$$

$$\vec{\nabla} \cdot \vec{v} = 0.0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

## Rotation and vorticity



Deformation.



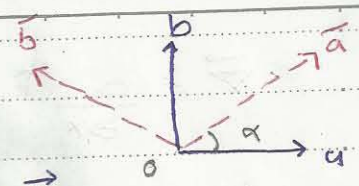
Rotation

The fluid Rotation  $\bar{\omega}$ , is average angular velocity of any two mutually perpendicular line element of the particles in each plane.





$$\vec{w} = w_x \hat{i} + w_y \hat{j} + w_z \hat{k}$$



$$\vec{\omega} = \frac{\vec{\omega}_{oa} + \vec{\omega}_{ob}}{2} = \frac{1}{2} \nabla \times \vec{v}$$
$$= \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$= \frac{1}{2} \left( \hat{i} \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) - \hat{j} \left( \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} \right) + \hat{k} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \right)$$

if  $\vec{\omega} = 0, 0 \rightarrow$  irrotational flow

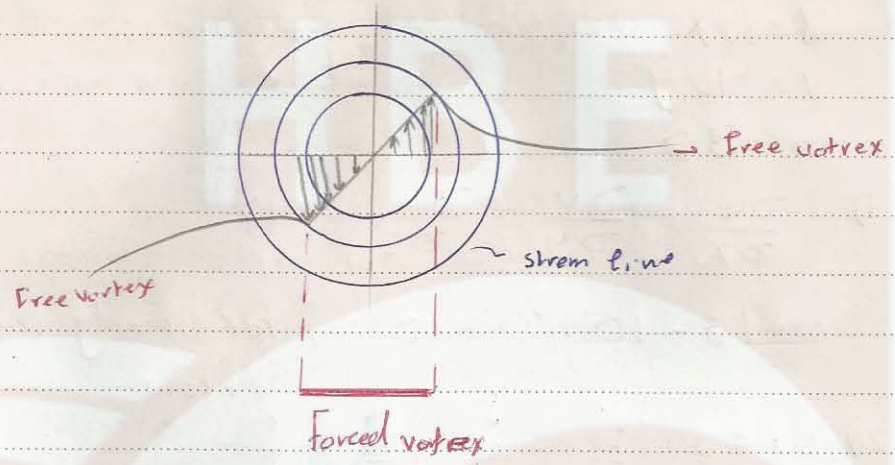
$\vec{\omega} \neq 0, 0 \rightarrow$  rotational flow

$$\text{vorticity} = \underset{\text{curl}}{2\omega} = \nabla \times \vec{v}$$



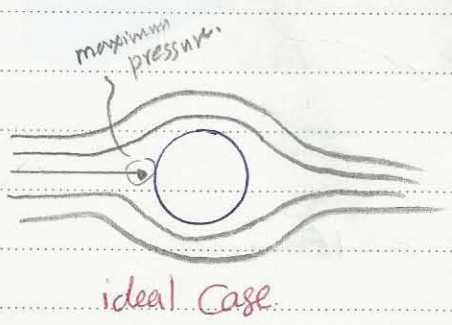
# \* (Vortex)

allow for which stream lines are concentric circles.

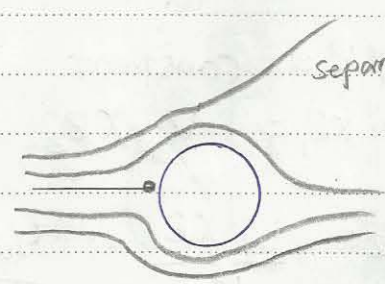


→ **Forced Vortex** :- is a vortex in which velocity is linearly proportional to the distance from the center of the vortex.

→ **Free vortex** :- is a vortex in which the product (V) velocity (R) radius is equal to constant



ideal case



separation

→ nature of up stream flow  
→ Boundary layer Rotation.





ex a flow is represented by the velocity field.

$$\vec{V} = 10x\hat{i} - 10y\hat{j} + 30\hat{k}, \text{ Determine the field}$$

- 1) possible incompressible flow.
- 2) irrotational.

→ solution:-

$$u = 10x$$

$$v = -10y$$

$$w = 30$$

$$1) \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.0$$

$$10 - 10 + 0 = 0, \text{ for steady incompressible flow.}$$

$$2) \omega = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 10x & -10y & 30 \end{vmatrix}$$

$$= \frac{1}{2} [\hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(0-0)] \Rightarrow 0 \rightarrow \text{irrotational flow.}$$

ex a free vortex has a velocity of 40 m/sec. at  $R = 4$  km from the vortex center. Find the velocity at 10 km from the center.

$$V \times R = \text{constant}$$

$$V_1 \times R_1 = V_2 \times R_2$$

$$V_2 = \frac{V_1 \times R_1}{R_2} = \frac{40 \times 4000}{10000} = 16 \text{ m/sec.}$$



## \* Pressure variation in flowing fluid.

- pressure difference below and above the wing → lift force.
- blood pressure is responsible on motion of blood.

CH.5

## \* Basic Causes for pressure variation:-

- 1) pressure variation due to (gravity) → pressure variation with elevation.

$$\frac{dp}{dz} = -\gamma$$

incompressible flow

$$dp = -\gamma dz$$

$$p + \gamma z = p = \text{constant.}$$

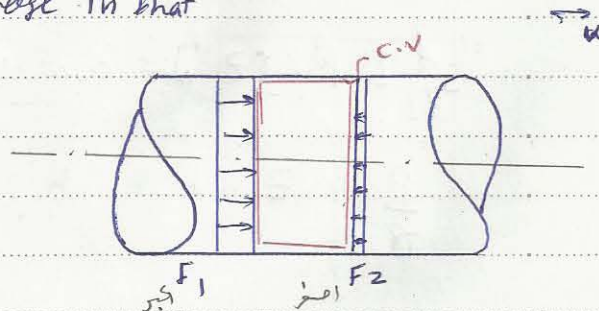
$$\frac{p}{\gamma} + z = h = \text{piezometric head.}$$

→ hydrostatic = no acceleration.

- 2) acceleration.
- 3) viscous resistance.

→ To accelerate a fluid in a certain direction there must be a net force in that direction.

→ The pressure must be decrease in that direction of flow.



$$F_1 = P_1 \cdot A_1 \quad F_2 = P_2 \cdot A_2$$





\* Pressure variation due to weight and acceleration.

$$\Delta W = \gamma \cdot \Delta V \quad \Rightarrow \quad W = \gamma \Delta L \Delta A$$

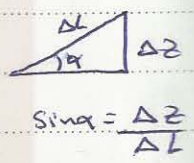
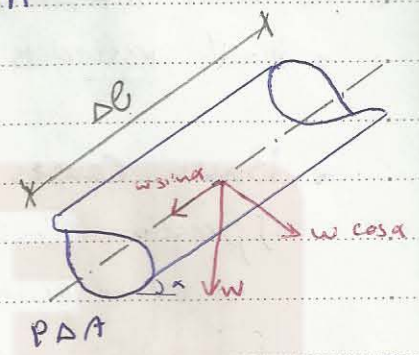
$$\Delta V = \Delta L \cdot \Delta A$$

$$\sum F_e = m a_e$$

$$P \Delta A - (P + \Delta P) \Delta A - \gamma \Delta L \cdot \Delta A \cdot \frac{\Delta z}{\Delta L} =$$

$$\frac{P \Delta A}{\Delta L} - \frac{(P + \Delta P) \Delta A}{\Delta L} - \gamma \Delta L a_e$$

$$\frac{-P}{\Delta L} - \gamma \frac{\Delta z}{\Delta L} = + \rho a_e$$



By taking the limit :-

$$\frac{-dP}{dL} - \gamma \frac{\Delta z}{\Delta L} = \rho a_e$$

$$\frac{-d}{dL} [P + \gamma z] = \rho a_e \Rightarrow \text{Euler's equ. for fluid in motion.}$$

For acceleration.

$$\frac{-d}{dL} [P + \gamma z] = 0, \Rightarrow P + \gamma z = \text{constant.}$$

$$\frac{P}{\gamma} + z = h$$

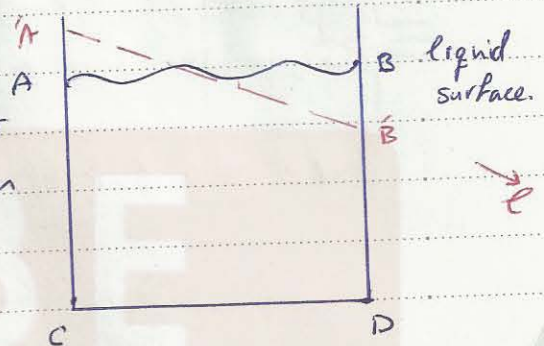




examples of pressure variation resulting from acceleration.

→ Uniform liquid tank:-

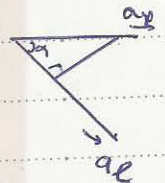
to accelerate this tank of fluid in the x-direction there must be an external force in the x-direction



→ applying Euler's eqn. along the line  $\hat{A}\hat{B}$  (x-direction)

$$-\frac{d}{d\ell} (P + \gamma z) = \rho \cdot a_\ell$$

$$\frac{dp}{d\ell} = 0 \rightarrow -\frac{d\gamma z}{d\ell} = \rho \cdot a_\ell = \rho a_x \cdot \cos \alpha$$



For incompressible fluid.  $\rho = \text{const.} \rightarrow \gamma = \text{const.}$

$$-\frac{dz}{d\ell} = \frac{a_x \cdot \cos \alpha}{g}$$

$$\sin \alpha = \frac{a_x}{g} \cos \alpha \Rightarrow \frac{\sin \alpha}{\cos \alpha} = \tan \alpha = \frac{a_x}{g}$$

→ Rotation of tank.

$$-\frac{d}{dr} [P + \gamma z] = \rho \cdot a_r$$

$$\frac{\partial}{\partial r} [P + \gamma z] = \rho \frac{\gamma^2 \omega^2}{r}$$

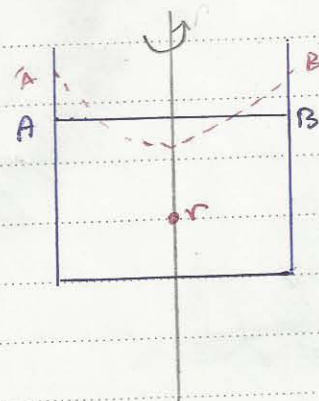
$$d[P + \gamma z] = \frac{\rho \gamma^2 \omega^2}{r} \cdot dr$$

بدماء

$$P + \gamma z = \frac{\rho r^2 \omega^2}{2} + C$$

$$P + \gamma z - \frac{1}{2} \rho r^2 \omega^2 = \text{const.}$$

$$P_1 + \gamma z_1 - \frac{1}{2} \rho r_1^2 \omega_1^2 = P_2 + \gamma z_2 - \frac{1}{2} \rho r_2^2 \omega_2^2$$



$$a_r = \frac{v^2}{r}$$

$$v = r\omega$$



18] Ex 13

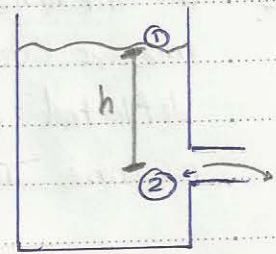


8 / July / 2012 Sun.

ex. Prove that  $v = \sqrt{2gh}$  for the following.

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2$$

$\frac{P_1}{\gamma}$  ← gauge pressure atmospheric  
 $\frac{v_1^2}{2g}$  ← (continuity eqn)  
 $\frac{P_2}{\gamma}$  ← atmospheric



$$V_1 \cdot A_1 = V_2 \cdot A_2 \Rightarrow \frac{V_2^2}{2g} = z_1 - z_2 \Rightarrow z_1 - z_2 = h$$

$$\frac{V_2^2}{2g} = h \Rightarrow V_2 = \sqrt{2gh} \quad \#$$

ex. what the pressure gradient is required to accelerate water  $\rho = 1000 \frac{\text{kg}}{\text{m}^3}$  in a horizontal pipe of Rate 6 m/s<sup>2</sup>.

Using Euler's method:-

$$-\frac{\partial}{\partial t}(P + \gamma z) = \rho a_x$$

$$-\frac{\partial P}{\partial t} - \gamma \frac{\partial z}{\partial t} = \rho a_t$$

horizontal

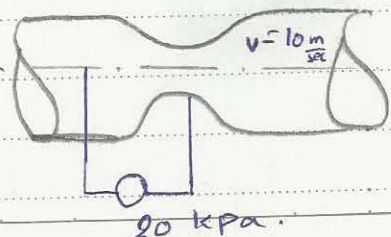
$$\frac{dp}{dt} = -1000(6) = -6000 \frac{\text{Pa}}{\text{m}}$$

p(5-59) ✓, p(5-60)

$$\text{ex. } \frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2$$

$$\frac{P_1 - P_2}{\gamma} = \frac{v_2^2 - v_1^2}{2g} \Rightarrow \frac{20 \times 10^3}{9810} = \frac{10^2 - v_1^2}{2 \times 9.81}$$

$$v_1 = 7.11 \text{ m/sec}$$



56

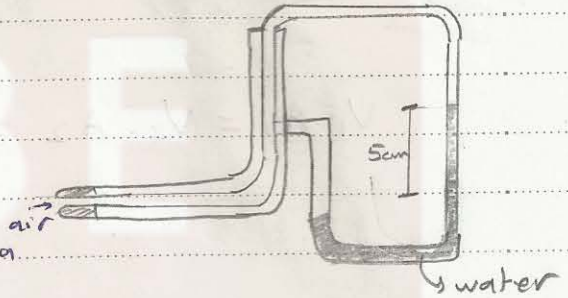


ex. an air manometer is connected to a pitot tube used to measure velocity of the air if the manometer is deflected by (5 cm) what is the velocity, assume  $T = 290 \text{ K}$ ,  $p = 105 \text{ kPa}$ .

$$v = \sqrt{\frac{2 \Delta p}{\rho}}$$

$$\Delta p = \rho_{H_2O} \times 0.05$$

$$= 9810 \times 0.05 = 490.5 \text{ Pa}$$

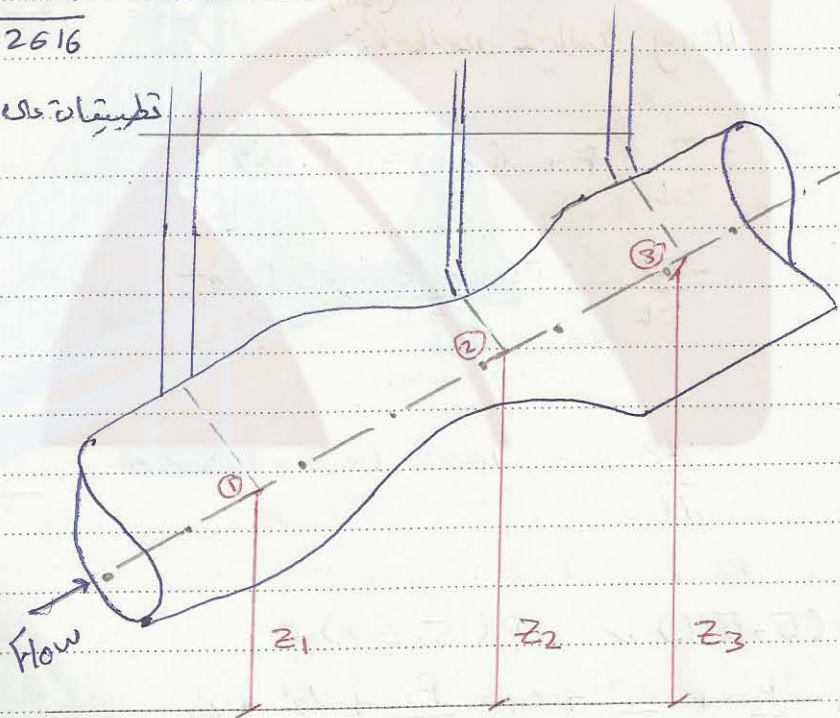


$$\rho = \frac{P}{RT} = \frac{105 \times 10^3}{287 \times 290} = 1.2616$$

$$v = \sqrt{\frac{2 (490.5)}{1.2616}} = 27.188 \text{ m/sec.}$$



تطبيق على برنولي







$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + Z_2 = \frac{P_3}{\rho} + \frac{V_3^2}{2g} + Z_3 \quad [m]$$

pressure head

velocity head

elevation head

∴  $P_1 = P_2 = P_3$

1) steady

2) incompressible

3) continuous

4) irrotational

5) no heat

6) no shaft

7) no vis cover

or

$$P_1 + \frac{1}{2} \rho V_1^2 + \rho g Z_1 = \text{constant} \quad [Pa]$$

\*

Venturii tube meter.

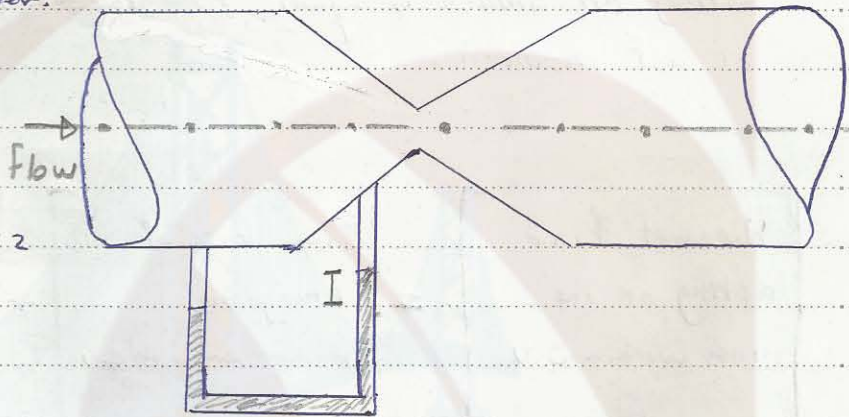
1) continuity eqn.

$$m_1 = m_2$$

$$\rho_1 \cdot V_1 \cdot A_1 = \rho_2 \cdot V_2 \cdot A_2$$

→ incompressible

$$\rho_1 = \rho_2$$



$$V_1 A_1 = V_2 A_2 \rightarrow V_1 = \frac{V_2 A_2}{A_1}$$

So, Bernouly eqn.

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \frac{P_1 - P_2}{\rho}$$





## Momentum Principle

B : extensive property

CH-6

$\beta$  : intensive property

$$B = \int \beta \, dm = \int \beta \rho \, dV$$

where  $B = m\bar{v}$

$$\beta = \frac{m\bar{v}}{m} = \bar{v}$$

$$\sum F = \frac{d(m\bar{v})}{dt} = \frac{d}{dt} \int_{C.V} \rho \bar{v} \, dV + \int_{C.S} \rho \bar{v} \cdot \bar{v} \, dA$$

→ Newton's second law:- The total summation of forces acting on a mass is equal to the rate of change of act moment.

$$\left[ \begin{array}{l} \text{the net force} \\ \text{acting on the} \\ \text{mass within C.V} \end{array} \right] = \left[ \begin{array}{l} \text{Time rate of} \\ \text{change on} \\ \text{momentum within C.V} \end{array} \right] + \left[ \begin{array}{l} \text{net out flow} \\ \text{of momentum} \\ \text{through the C.S} \end{array} \right]$$

$$\sum F = \sum F_B + \sum F_S$$

body force:  $\downarrow$

↳ surface force.

the force act through  
out the volume of the  
body [Gravitational and  
electromagnetic]

the force transmitted through the C.S  
① Pressure force, ② force transmitted  
through solid surface.





\* For steady state condition.

$$\frac{d}{dt} \int_{c.v} \rho v dV = 0.0$$

$$\Sigma F = \int_{c.s} \rho \vec{v} \cdot \vec{v} \cdot dA \rightarrow \text{variable velocity distribution across the flow passage.}$$

$$\Sigma F = \Sigma \rho \vec{v} \cdot \vec{v} \cdot A \rightarrow \text{uniform velocity distribution across the flow passage.}$$
$$= \Sigma m_{out} v_{out} - \Sigma m_{in} v_{in}$$

\* momentum eqn. in the cartesian coordinate system \*

$$\Sigma F_x = \frac{d}{dt} \int_{c.v} \rho v_x dV + \int_{c.s} \rho v_x v \cdot dA$$

$$\Sigma F_y = \frac{d}{dt} \int_{c.v} \rho v_y dV + \int_{c.s} \rho v_y v \cdot dA$$

$$\Sigma F_z = \frac{d}{dt} \int_{c.v} \rho v_z dV + \Sigma m_{out} v_{out}(z) - \Sigma m_{in} v_{in}(z)$$

\* Jet Deflected a plate or van

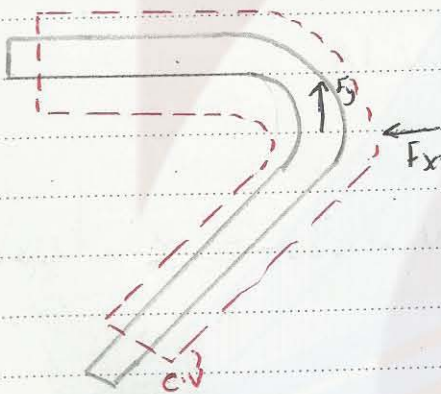
assumption:-

1) pressure of the <sup>inlet</sup> stream entering the plate is equal to the stream pressure leaving it.

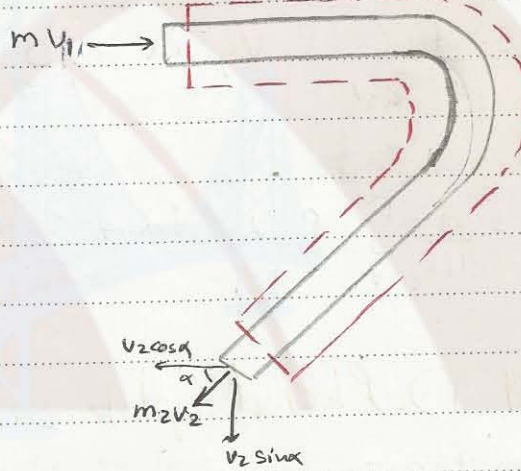
2) the velocity of the jet is not appreciably changed.

Fluid

خطوة في رفع الامن



Force diagram.



Momentum Diagram

(x-momentum)

$$\sum F_x = \frac{d}{dt} \int_{c.v} \rho V_x dx + \sum m_{out} \cdot V_{out}(x) - \sum m_{in} V_{in}(x)$$

Steady state condition





$$\begin{aligned} -F_x &= m_{out} V_{out}(x) - m_{in} V_{in}(x) \\ &= \rho Q [V_{out}(x) - V_{in}(x)] \end{aligned}$$

$$\begin{aligned} m_{in} &= m_{out} \\ &= m = \rho Q \end{aligned}$$

$$+F_x = \rho Q [V_2 \cos \alpha - V_1]$$

$$F_x = \rho Q [V_2 \cos \alpha + V_1]$$

(y = momentum)

$$F_y = m_{out} V_{out}(y) - m_{in} V_{in}(y)$$

$\swarrow$  zero

$$F_y = \rho Q [-V_2 \sin \alpha]$$

So, the assumed direction not correct, so  $F_y$  is down ward direction

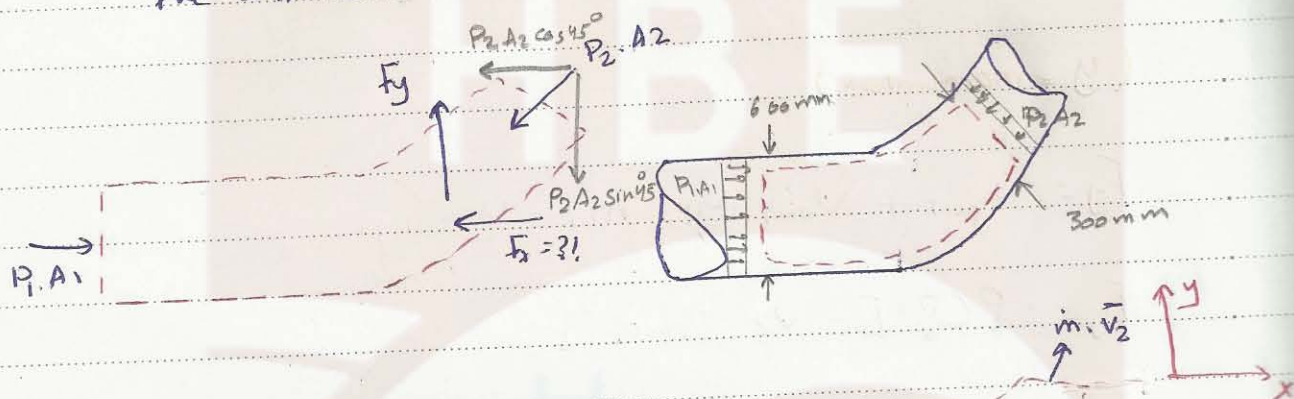
P(6-20)

[8.7] kids



10 / July / 2012

ex 1. A  $45^\circ$  reducing pipe-bend is a horizontal plane tapers from 600 mm diameter at inlet to 300 mm diameter at outlet. The gage pressure at inlet is 140 kPa and the Rate of flow of (H<sub>2</sub>O) through the bend  $0.1425 \frac{m^3}{sec}$  neglecting friction, calculate the Force acting by the water on the bend.



(x-momentum)

$mV_1$

$$\sum F_x = \frac{d}{dt} \int_{c.v} \rho V_x dx + \sum m_{out} V_{out}(x) - \sum m_{in} V_{in}(x)$$

0 = steady state condition.

$$\dot{m} = Q = m_{in} = m_{out}$$

$$P_1 A_1 - P_2 A_2 \cos 45 - F_x = m_{out} V_{2x} - m_{in} V_{1x}$$

$$P_1 A_1 - P_2 A_2 \cos 45 - F_x = \rho Q [V_2 \cos 45 - V_1] \rightarrow (1)$$

$$V_1 = \frac{0.1425}{\left(\frac{\pi}{4}\right)(0.6)^2} = 1.5 \text{ m/sec.}$$

$$V_2 = \frac{0.1425}{\left(\frac{\pi}{4}\right)(0.3)^2} = 6 \text{ m/sec}$$





$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \left\{ \begin{array}{l} z_1, z_2 \\ \text{horizontal} \end{array} \right.$$

$$\frac{140 \times 10^3}{9810} + \frac{1.15^2}{19.62} = \frac{P_2}{9810} + \frac{6^2}{19.62}$$

$$P_2 = 123,125 \text{ kPa.}$$

معوض في الاتجاه ①

$$F_x = 32265 \text{ N.}$$

(y-momentum)

$$-P_2 \cdot A_2 \cdot \sin 45^\circ + F_y = m_{out} \sin 45^\circ - \text{Zero}$$

$$123,125 \times 10^3 \times 0.707 \times \frac{\pi}{4} (0.13^2) - 1000 \times 0.425 \times 0.707 = F_y$$

$$F_y = 6123,11 \text{ N.}$$

p(6,13) and p(6,26)